A STUDENT'S MANUAL

OF A

LABORATORY COURSE

IN

PHYSICAL MEASUREMENTS.

BY

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PREFACE.

THIS manual was primarily written for use in the course in Harvard College known as "Physics C," and the experiments here detailed are based upon those performed in that course. The same course is given in the Summer School of Harvard University, beginning usually the first week in July and continuing six weeks, attended mainly by teachers. As given in college it requires 180 hours spent in the laboratory, and the student is held accountable for 90 hours of outside study. A knowledge of algebra and plane geometry, and a slight acquaintance with the notation of trigonometry is necessary. It should properly be preceded also by a more elementary course in physics, either by laboratory or text-book, preferably the former. For this purpose is recommended the quantitative course outlined in A Text-Book of Physics, Hall & Bergen, Holt & Co., New York. This course has for some time been in use most successfully as the laboratory alternative of the elementary physics required for admission to Harvard College. The course outlined in the following pages is designed to immediately follow this in college and to fit for the more advanced courses. It corresponds also to what is known as the advanced admission requirements in physics.

This manual, intended for students' use, has been given the form of an abstract of the daily lectures preceding the laboratory work and describing the experiments to be performed. It is intentionally condensed. A more extended treatment would render the book unwieldy and inconvenient for ready reference, and would perhaps allow the laboratory work to be a thoughtless following of too complete directions.

Many colleges have apparently been deterred from the more liberal introduction of the laboratory method of teaching physics by the great expense involved in fitting a laboratory for classes of any considerable size. The high price of apparatus to be found on the market allows even a fairly well endowed institution to purchase at most but a few instruments of each kind. But this, while in a certain way economical in the initial expense of apparatus, is very extravagant of teaching force, making an instructor wisely hesitate before assuming the burden of extending the course to his larger classes. For, with but few duplicate pieces of apparatus, either the laboratory sections must be small and the hours of work numerous, or one student must be engaged on one experiment while another is at work on some other, involving thus a separate detailed explanation to each. The departure from a systematic order of experiments, which this rotation method necessitates, is fatal to any progressive development of ideas. Moreover, apparatus designed originally for research or advanced work is often too delicate to allow that handling and inspection which is in itself most instructive. The complexity which adapts it to its original purpose unfits it for use as a class-room instrument, tending rather to confuse and perplex the student. In starting a laboratory course much routine detail work is involved in the devising of small pieces which it has been impossible to find on the market in the exact form required. Much of this must be home made, and the labor of superintendence is great. To remove as far as possible these difficulties a set of instruments has been devised especially adapted to the requirements of this course. The designs have been placed in the hands of instrument makers in Boston — Gillis & Gleeson, 106 Sudbury St. — with the restriction merely that the apparatus, when put upon the market, shall be sold at a price but little more than sufficient to pay for its proper manufacture and storage. The Physical Department of the University, being in no wise pecuniarily interested in

the manufacture of the apparatus, is not responsible for its mechanical perfection. If, however, this is at any time at fault, information to that effect will be considered an especial favor. No monopoly has been given for the manufacture of this apparatus, only the opportunity for that monopoly which an instrument maker can create for himself by good workmanmanship and reasonable charges.

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INTRODUCTION.

An effort is here made to explain all the corrections to be applied, and call attention to all the precautions which should be taken in the accurate and proper performance of the experiments. On the other hand, in the majority of cases, the description is purposely not such as will admit of a mechanical and unintelligent interpretation.

A precaution once noted is not again mentioned, but should always be taken wherever applicable. The number of observations which should be made in each case is not specified, but in regard to this the following may be of interest: — One observation has no check against blunders and as a rule contains in itself no criterion of its accuracy. A greater number of observations supplies this deficiency, and diminishes the probable error of the mean. Errors of observation, and other errors as liable to be above as below the true value, are alone diminished in this way. Instrumental errors cannot thus be eliminated except by changing in each observation the conditions of the experiment — for example, in determining the zero of the spherometer, by using first one side and then the other of the plate glass, and by using different parts of the surface; or in the experiment with the spectrometer, by using different parts of the graduated circle. By taking a large number of observations therefore the student will not only become more familiar with the principles of the experiment and the handling of the apparatus, but will also increase the accuracy of his results.

While accuracy in measurement is desirable, precision of statement is no less so, and as a violation of this must be included the disregard of the number of figures which are significant. If three measurements of the length of a rod are 12.67, 12.64 and 12.66 cm., their sum is 37.97. The student, on dividing this number by three to find the average, might write it 12.65666 indefinitely, as if the mere process of dividing by three would enable him to measure to the ten and hundred thousandth of a centimeter. Of this number only the first four figures are significant, and to keep more is claiming an accuracy unattained. The same mistake is often made in multiplying, the percentage error of the least accurate factor determining in general the percentage error of the result. For example, it is desired to find the volume of a rod, and rough measurements have been made of its dimensions, giving for the length 498. cm. and for its diameter 2.4 cm. The calculation of the volume of the rod based on these measurements would give

$$V = l\pi r^2 = 498. \times 3.1 \left(\frac{2.4}{2.}\right)^2 = 2223.072 \text{ cu. cm.}$$

In truth, however, only the first two figures are known and the result should be written 2200 cu. cm. If now greater care is taken in measuring and the diameter of the rod is found to be 2.37 cm., the student is justified in keeping another figure in the result and it may be written 2190 cu. cm. Still further care in measurement might justify keeping still another figure in the result. The record should, on the other hand, show all the accuracy attained. If several succeeding figures are known to be ciphers they should be entered as such although decimals. For example, if in weighing with balances sufficiently sensitive to detect milligrammes, the weight of a body is exactly 12 grammes, it should be entered as 12.000 grammes.

It is well to note in this connection that, while the final result should have the highest degree of accuracy attainable,

it does not follow that every factor should be determined with the greatest possible degree of accuracy. For example in the above case it is wholly unnecessary to determine the length of the rod to tenths of a centimeter, although with care it may readily be done. In fact so limited is the accuracy of the other factor that the result would not be affected by an error of a whole centimeter in the measurement of the length. Whatever time can be saved at this point might profitably be devoted to the determination of the diameter of the rod with more care and greater number of observations.

Forms and blanks for each experiment, to be filled by the student, are not recommended for the keeping of notes. Some system however is desirable and should cover the following points — date, object of experiment, apparatus (name and number), method, data, calculation and results. Under the head of data should be entered all observations however seemingly unimportant. For instance in weighing, the separate excursions of the pointer to right and left should always be preserved.

Observations should not be entered at random over the page, but according to a carefully planned system; and each should be accompanied by a memorandum so full that another person in reviewing the notes could at a glance determine its meaning, of what it is a measure, and in what units it is expressed. This above all things is essential. It is desirable that the observations and calculations should admit of ready comparison one part with another. The notes should therefore be compact and well arranged, and the writing and figuring closely spaced.

In order to avoid continual repetition in the following pages all measurements will be assumed to be in the centimeter-gramme-second (c. g. s.) system, and temperature observations will be on the centigrade scale.

In higher experimental work, whether research or practical, the investigator is determining a quantity which is to him

unknown. It is desirable to reproduce this condition, as far as possible, in an elementary laboratory course. Therefore the student should not ask himself whether his results are correct and in agreement with known values, but whether his method is correct and his observations well taken. Neither text-books, tables of physical constants, nor the instructor should be approached with this question, but the student as the observer should decide for himself.

Whether the observations are used or discarded they should all be preserved, and if discarded in the calculation the reason should be entered in the notes. The only observations which need not be preserved are those which the observer has taken for the sake of practice with the express intention of afterwards discarding them.

EXPERIMENTS.

MECHANICS.

1. VERNIER GAUGE.

The vernier gauge, an instrument for the measurement of length, is provided with two jaws between which is placed the object to be measured. One jaw is permanently fixed to a metric scale upon which the other slides. On the movable jaw is a mark which should coincide with the zero of the scale when the jaws are together; and when the object to be measured is placed between them this line marks on the scale their distance apart, and thus indicates the length of the object. After re-

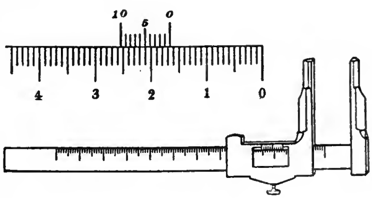


FIG. 1.

cording the number of whole centimeters and millimeters, if a fraction of a millimeter remains it also may be read off directly and accurately by means of a subsidiary scale called a vernier. This is a small scale graduated upon the movable jaw, beginning with the mark above mentioned. If the vernier is to read to tenths of a millimeter against a millimeter scale the vernier is nine millimeters long, but is subdivided into ten parts. Each division of the vernier therefore is shorter than the division opposite it on the main scale by one-tenth of a millimeter. Along the vernier each succeeding line will be nearer its corresponding line on the main scale by one-tenth of a millimeter. Thus the number of the vernier line, which exactly coincides with a line on the main scale, will be the value in tenths of a millimeter of the fraction which is being measured. In the diagram the reading is 1.66 centimeters.

A more careful examination of the vernier will give some information also in regard to the next decimal place.

The zero error to be applied as a correction to all measurements should be carefully determined by taking the reading when the jaws are pressed gently together. A plus or a minus sign should be prefixed to indicate whether the correction is one to be added or subtracted. Holding the instrument to the light note at what point the jaws touch, and use this part of the gauge in subsequent measurements.

Measure nine diameters of a large glass ball, and calculate its volume, $V\,(\text{sphere}) = \frac{4}{3}\pi r^3$. Measure the length and several diameters of an aluminum cylinder, and calculate its volume, $V\,(\text{cylinder}) = l\pi r^2$.

In regard to the number of figures which should be retained in the result, see Appendix I.

2. MICROMETER GAUGE.

The micrometer gauge, although not as convenient as the vernier gauge and incapable of measuring as great lengths, is nevertheless a more accurate instrument. It consists of a screw moving in a nut toward or away from a fixed stop. One complete turn of the screw advances or withdraws its end by an amount equal to the distance between the threads of the screw — a fraction of a turn by a proportional amount. The number of whole turns may be read on a longitudinal scale, and the fraction of a turn by graduations upon the head of the screw. The object to be measured is placed between the stop and the end of the screw, and the latter turned down upon it. The length of the object is then read off directly in units which depend on the pitch of

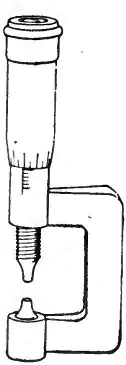


FIG. 2.

the screw. Determine the zero error, before and after measuring, by taking the readings with the jaws closed. Apply this, with the proper sign, to all readings. If the instrument is provided with a friction or ratchet head always hold by it in turning down the screw, thus securing uniform pressure. If a ratchet head is being used it should be turned until the ratchet just begins to act, but it should not be allowed to turn further as the pounding would increase the pressure.

Measure three mutually perpendicular diameters of each of seven steel bicycle bearing balls, and calculate the total volume of the balls.

3. SPHEROMETER: RADIUS OF CURVATURE.

A spherometer, in its essential features, consists of a screw moving in a nut mounted on three legs. It is a form of micrometer in which the stop is replaced by the plane passing through the points of the three legs. The zero reading may be obtained by placing the instrument on a clean piece of plane plate-glass and turning down the center screw until its point just touches the surface lightly. Repeat several times on each side of the glass in order to correct for possible warping.

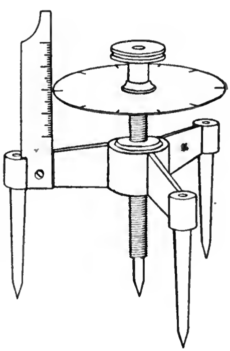


FIG. 3.

Place some thin object (a microscope cover glass) upon the plate glass and lower the screw upon it. The difference between this reading and the zero reading is the thickness of the object in units which are determined by the pitch of the screw.

Place the instrument upon a spherically convex surface and read. If we call a the difference between this and the zero, and d the average distance from the center screw to the

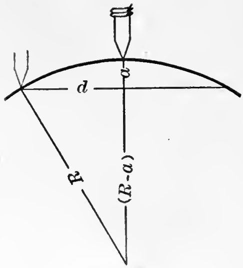


FIG. 4.

three legs, the accompanying diagram will represent the arrangement. By geometry:

$$R^2 = (R - a)^2 + d^2$$
$$R^2 = R^2 - 2Ra + a^2 + d^2$$
$$R = \frac{a^2 + d^2}{2a}$$

d may be obtained by making an impression of the four points upon a piece of paper placed upon a plane hard surface, and measuring the distances with a vernier gauge. Measure separately the radius of curvature of each surface of a double convex lens.

4. NICHOLSON'S HYDROMETER.

Nicholson's hydrometer is designed for use as a hydrostatic balance in determining the weight and density of a solid. Preliminary to this use however it is necessary to determine the weight which added to the upper scale pan will sink the instrument in water to a given mark on the stem. This depends upon the temperature, for the principle of the instrument is that the weight of a floating body is equal to the weight of the liquid displaced, and on an increase of temperature both the metal of the hydrometer and the water expand, the expansion of the former tending to increase the volume and that of the latter to diminish the density of the displaced water. The two expand unequally. As a result the weight of the displaced liquid varies, and as the weight of the instrument must be equal to this, the weight added to the upper scale pan must be varied accordingly.

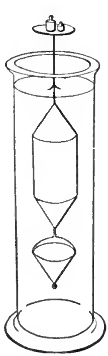


FIG. 5.

Find the weight which must be added to the upper scale pan in order to sink the hydrometer to a given mark on the stem in water at about the temperatures 5, 10, 15, 20, 25 and 30 degrees centigrade. Fractions of centigrammes may be estimated by interpolation. Plat the results on coördinate paper, taking temperatures for abscissae and weights for ordinates. (See Appendix II.)

Care must be taken that the hydrometer is freed from all adhering air bubbles, that it does not touch the sides of the vessel, and that the upper scale pan is dry. The last two conditions may be secured by guiding the stem through a torn paper placed around it and resting upon the top of the jar. The stem of the instrument should be freed from oil by careful cleaning with a weak solution of caustic potash (or of soap), thereby reducing to a minimum the irregularity of the capillary action of the water on the stem. The water should be kept thoroughly stirred that it may be of uniform temperature throughout. In judging the level of the water on the stem look through the water, sighting along the under surface.

5. SPECIFIC GRAVITY OF A SOLID BY NICHOLSON'S HYDROMETER.

The specific gravity of a solid is the ratio of its weight to the weight of an equal volume of water. It may be determined by means of the hydrometer, by weighing the solid first in the upper scale pan, and then when submerged in water and resting on the lower pan. When submerged it weighs less, being lifted in part by the water. The loss of weight when in water is equal to the weight of an equal volume of water.

Note the temperature of the water, and determine from the curve of the last experiment the total weight, which added to the upper scale pan of the hydrometer sinks it to the mark on the stem. Place the object to be weighed, a brass disc, in

the upper scale pan and add weights until the hydrometer sinks to the mark on the stem. The total weight in the upper scale pan has just been determined from the curve. By subtracting from this the part which is known, determine the weight of the brass disc. Place the disc on the lower scale pan, again add weights to the upper scale pan, and repeating the above process, determine the weight in water of the brass disc. Subtract from the weight of the disc its weight when submerged in water, — the difference is the weight of an equal volume of water. Divide the weight of the brass disc by the weight of the water and find the specific gravity of the brass.

In the centimeter gramme system this result will also be equal numerically to the number of grammes of brass occupying one cubic centimeter of space.

Strictly the above experiment should have been performed in water at a temperature of 4° C., but it is so difficult to secure this temperature and to maintain it constant, that it is far better to have the water at nearly the temperature of the room. The error introduced in this experiment is of barely perceptible magnitude.

6. READING BY VIBRATIONS.

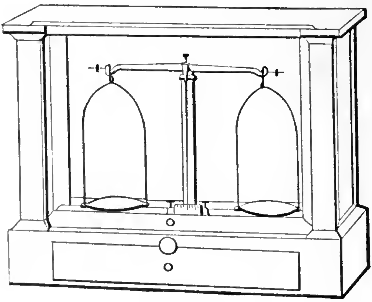


FIG. 6.

Since time is consumed in waiting for the pointer of an ordinary arm balance to cease swinging, it is best to determine the point at which it would come to rest by observing its turning points. The mean between the average of the readings to the right and the average of

the readings to the left of several consecutive swings will be the point of rest. To correct for the error due to the diminution of the amplitude of swing by friction take an uneven number of swings, thus throwing both the greatest and the least upon the same side. If the pointer moves over a small scale numbered from 0 to 20, suppose the consecutive excursions are as follows:—

To left.	To right.
8.6	
	12.6
8.9	
	12.2
9.3	
3) 26.8	2) 24.8
8.9	12.4
	8.9
	2) 21.3
	10.6 point of rest.

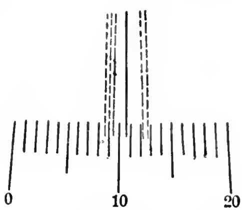


Fig. 7.

Determine the zero point (*i.e.* with no load in either scale pan) by means, separately, of three, five, and seven swings with an amplitude (from the one side to the other) of about three divisions. Then once (five swings) with an amplitude of six or seven divisions, and again with one of nine or ten. Finally, allow the pointer to come to rest and note its final position.

The balance should be so leveled upon the table that it cannot rock and thus change the zero point during the progress of the experiment. Disturbing air currents, which will make the balance swing irregularly, may be avoided by always lowering the front of the case while weighing. As the pointer is some distance in front of the scale its reading will depend somewhat upon the position of the observer. Error from this source may be avoided by placing a small mirror behind the pointer, and always taking such a position that the latter hides its own image.

The zero point is to be redetermined each day before beginning work with the balance; usually two sets of observations are sufficient.

7. SENSITIVENESS OF A BALANCE.

The sensitiveness of a balance depends upon the weight of the balanced system and upon the distance of its center of gravity below the central knife edge. It varies with different loads not only on account of the varying weight but also on account of the displacement of the center of gravity and the bending of the arms. It is measured by the number of scale divisions through which the pointer is moved by the addition of one centigramme (or milligramme) to one scale pan.

Determine the position of rest with no load in either scale pan by the method of vibrations. Add one centigramme to one scale pan and determine the new point of rest in the same way. The difference between the two will be the sensitiveness of the balance for zero load. Find the point of rest for a load of one gramme in each scale pan, and then with an additional weight of one centigramme on one side. The difference between the readings is the sensitiveness for a load of one gramme. Determine the sensitiveness for zero load, and for loads of one, five, ten, twenty, thirty, forty and fifty grammes. Plat the results on coördinate paper, the loads as abscissae and the corresponding sensitiveness as ordinates.

In adding or removing weights the scale pans should always be lowered until they rest upon the bottom of the case, as otherwise the knife edges will soon become dulled. The weights should be handled by means of pinchers.

8. DOUBLE WEIGHING.

In accurate work with balances it is necessary to correct for inequality in the lengths of the arms. The ratio of the

lengths may be determined and the correction applied by the following method. Let the lengths of the arms be a (left), and b (right). Place some unknown body x (over fifty grammes) in the left scale pan. By trial determine what weight in the opposite scale pan will bring the pointer nearest to the true zero point previously obtained, and determine the point of rest by vibrations. If this is not exactly the zero, balance is not quite obtained. Add one centigramme. This will perhaps send the pointer to the other side of the zero. One weight is too small, the other is too large, but the exact weight required can be found by the following process of interpolation. The difference between the two readings (the distance that the pointer is moved by one centigramme) is the sensitiveness of the balance; its reciprocal is the fraction of a centigramme required to move the pointer one division; the latter multiplied by the distance from the first point of rest to the zero point is the fraction of a centigramme which should have been added to secure exact balance: for example, if the zero point is 10.5, and if the point of rest is 11.2 when the weight is 55.16 grm., and 9.8 when the weight is 55.17 grm., one centigramme moves the pointer 1.4 divisions, and the weight necessary to move it 1 division is $\frac{1}{1.4}$ of a centigramme; and the weight necessary to move it from 11.2 to 10.5, that is .7 of a division, is $\frac{.7}{1.4}=.5$ of a centigramme, giving for the true weight necessary to secure balance $w_1=55.165$ grm. This method, called interpolation, makes it possible to weigh to milligrammes although the smallest weight in the set is a centigramme.

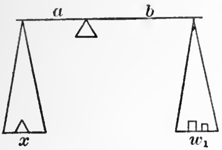


FIG. 8.

From the principle of the lever we have

$$ax=w_1b.$$

Change x to the opposite scale pan and determine the weight w_2 necessary to exactly balance it. The weights now are on the arm (a) and the equation becomes

$$bx = w_2 a\,.$$

Multiply one equation by the other.

$$abx^2 = w_1 w_2 ab\,.$$

Whence
$$x = \sqrt{w_1 w_2}\,,$$
or
$$x = \frac{w_1 + w_2}{2}$$

approximately since w_1 and w_2 differ but slightly. Calculate x.

Divide one equation by the other and we have

$$\frac{a}{b} = \frac{w_1 b}{w_2 a}\,,$$

or

$$\frac{a}{b} = \sqrt{\frac{w_1}{w_2}}\,.$$

From this calculate the ratio of the lengths of the arms.

This process of weighing a body, first in one scale pan and then in the other to eliminate the error arising from the inequality in the lengths of the arms of the balance, is called "double weighing."

9. WEIGHT IN VACUO.

The operation of weighing consists, strictly, in finding the number of standard masses called grammes which are attracted by the earth with as much force as is the body which is being weighed. If a weighing has been made in air by means of the "equal arm balance," it does not follow that the earth is attracting with equal force the weights and the body which is being weighed, even after a correction has been applied by

double weighing for any accidental inequality in the lengths of the arms. The experiment is complicated by the buoyant action of the air which tends to lift not only the body which is being weighed but also the brass weights. Properly therefore the experiment should be performed in vacuo. It is, however, usually more convenient to work in the air and then apply the necessary corrections. The buoyant force of the air, the lifting force which it exerts, is equal to the weight of the air displaced.

Weigh in air the aluminum cylinder of Exp. 1 by vibrations, interpolation and double weighing, and record the temperature of the air and the barometric pressure [and at times the humidity].

It is desired to find from this result the number of grammes which would have balanced the cylinder had the experiment been performed in vacuo. For a moment regard only the buoyant force exerted by the air on the brass weights. Had the experiment been performed in vacuo not so many weights would have been required since they would not be lifted in part by the air. The first correction therefore is to *subtract* from the above result the weight of the air displaced by the brass. To find the value of this correction calculate first the volume of the brass by dividing the number of grammes by the number of grammes required to occupy one cubic centimeter of space, (see the result of Exp. 5). Multiply this, the volume of the air displaced, by the weight of each cubic centimeter of air as determined from the table (1) given in Appendix III. The result is the correction to be subtracted. On the other hand, however, the cylinder weighed has also been buoyed up by the air, and consequently appears lighter than when weighed in vacuo. A second correction must therefore be applied by *adding* the weight of the air displaced by the cylinder. The volume of the aluminum cylinder is known from Exp. 1. Calculate the weight of the air which is displaced and add as the second correction.

The result thus corrected is the number of grammes which are attracted by the earth with the same force as is the aluminum cylinder. This is the knowledge that is almost always sought in weighing; and strictly no accurate weighing is complete until these corrections have been applied. Ordinarily the buoyancy of the air on the small fractional weights may be neglected. If the object being weighed is of irregular shape a special method, as explained in the next experiment, must be employed to find its volume.

Incidentally calculate the number of brass gramme weights which will displace a volume of air weighing one milligramme under the atmospheric conditions existing at the time of the experiment.

10. DENSITY OF A SOLID BY SUBMERSION.

The density of any substance is its mass (sometimes loosely called matter) per unit volume. Thus the mean density of any body is found by dividing its mass by its volume, $D = \frac{M}{V}$.

The unit of mass is the mass contained in the standard piece of metal called a gramme. The mass of any body may be most conveniently measured in terms of these units by determining the number of grammes which are attracted by the earth with the same force, for this attraction is proportional to the masses of the bodies acted upon. When the forces are equal the masses are equal, and are known, for the "weights" have their masses stamped upon them. This process of determining mass by the force of the attraction of the earth is called weighing and was explained in detail in the last experiment. By that experiment the mass of the aluminum was found, but the process of obtaining it assumed in one of the corrections that the volume of the cylinder was known. If the volume is not known, and the body is often

of such irregular shape that it cannot be determined by direct measurement, the following process gives first the volume approximately and then finally the volume far more accurately than it can be measured by the vernier gauge even under the most favorable conditions as to shape. This process then furnishes the denominator of the density fraction. In the following experiment work with the same aluminum cylinder, but wholly disregard any previous knowledge of its volume or its weight.

Weigh the aluminum cylinder by vibrations, interpolation and double weighing. A correction for the buoyancy of the air on the weights is to be immediately applied to the result of this weighing by subtracting one milligramme for every seven grammes of brass. This correction is not only to be applied to all weighings throughout this experiment but to all future weighings performed in air and justifying this degree of care. Tie around the cylinder a harness of fine iron wire and suspend it from the top of the stirrup of the balance in a beaker of water placed on a bridge over one of the scale pans. Weigh to milligrammes by interpolation, and record the temperature of the water. While being weighed the cylinder should be wholly submerged by the water and only one strand of wire should pass through the surface. Remove the cylinder and weigh the wire harness, adding water until the wire is submerged to the same level. The difference between the last two weighings will be the weight of the cylinder in water. Subtract the weight of the cylinder in water from its weight in air and the result will be approximately the weight of the water displaced, and in the centimeter gramme system this will be an *approximate* volume. With this find the weight of the air displaced by the cylinder as in the last experiment, and add it to the weight of the cylinder in air. This will give the weight of the cylinder in vacuo, numerically equal to its mass, and the numerator of the density fraction. Now subtract from the

weight in vacuo the weight in water and the result will be the exact weight of the water displaced. Multiply this by the volume of each gramme of water. (Appendix III, table 2.) The result will be the exact volume of the water displaced, therefore also the volume of the cylinder and the denominator of the density fraction.

Calculate from this the mean density of the aluminum.

11. DENSITY OF A LIQUID BY DISPLACEMENT.

A similar process may be employed in determining the density of a liquid.

Weigh the cylinder of the preceding experiment submerged in the liquid, and correct for the weight of the wire harness. Subtract the weight of the cylinder in the liquid from its weight in vacuo; this gives the weight of the displaced liquid. Its volume is equal to the volume of the cylinder. From these its density may be calculated.

Determine by this method the density of alcohol (or kerosene or glycerine), borrowing all the data possible from the last experiment. Make a note of the temperature of the liquid.

12. CAPACITY OF A SPECIFIC GRAVITY BOTTLE.

Find the capacity of a glass stoppered specific gravity bottle by weighing empty (clean and dry); and then full of water at a known temperature. The difference is equal to the weight of the water in the air and, in the c. g. s. system, to its approximate volume. Taking the temperature and the barometric pressure find the weight of air displaced by the water. Adding this to the weight of the water in air find the weight of the water in vacuo. From this find, as in a former experiment, the exact volume of the water. This will be the desired capacity of the bottle.

13. DENSITY OF A LIQUID BY THE SPECIFIC GRAVITY BOTTLE.

Weigh the bottle used in the last experiment when filled with alcohol. Subtract from this the weight in air of the bottle alone, thus obtaining the weight in air of the alcohol. Applying the principles of the preceding experiment, and making all the proper corrections find the density of the alcohol.

14. DENSITY OF A SOLID BY THE SPECIFIC GRAVITY BOTTLE.

Weigh a number of pieces of the solid, sufficient in amount to nearly fill the specific gravity bottle used in Exp. 12. Place in the bottle; fill with water, and reweigh. Borrow the necessary data from the previous experiment. The difference between the weight of the bottle filled with water and the solid, and the bottle filled with water alone, is what has been called the weight of the solid in water. From this, together with the weight of the solid in air, there may be obtained — in order — the approximate weight of water displaced, the weight of the air displaced by the solid when weighed in air, the weight of the solid in vacuo, the exact weight of the water, and the true volume, and from these data the density of the solid. Principles already explained and used in preceding experiments are alone involved.

Determine the density of drawn aluminum wire by this method.

15. MEASUREMENT OF SURFACE TENSION BY CAPILLARY ACTION.

Throughout the interior of a liquid each particle is attracted equally in all directions by the immediately adjacent

particles. At the boundary of the liquid, however, one of two phenomena will occur. If the boundary of the liquid is formed by its contact with a solid, the particles of the liquid very close to the surface are attracted more by the solid than by the liquid within. As a result the pressure of the liquid in the immediate neighborhood of the solid is increased, and the liquid tends to spread itself over the surface of the solid. If, on the other hand, the bounding surface is formed by contact with air, the particles of the liquid near the surface are attracted less by the air than by the liquid within, and the surface tends to contract—that is, there results a surface tension. In the first case there is a surface pressure—a tendency of the surface to extend; in the second case there is a surface tension, and as a result the surface tends to contract. These two effects together give rise to the phenomenon of capillarity. If the solid is a tube of small bore partly immersed, vertically, the liquid will spread up the interior surface of the tube. As it does so it will expose a long concave surface to the air, which will tend to contract, and will thus draw up the liquid in the tube. The liquid will continue to rise until the weight of the liquid lifted is equal to the lifting force. The surface of the liquid exposed to the air will tend to contract into a plane, but as it is lifted by its edges and dragged down at its center by the weight of the liquid it will, in its final state, be still concave. It has been shown by experiment that if the tube is of glass, and if the liquid is water, the *free* surface of the water will make an angle of 25° 32′ with the surface of the glass.

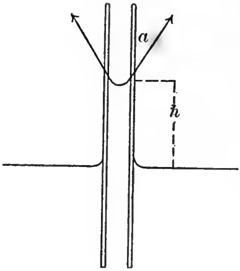


FIG. 9.

The tension across a line, one centimeter long, drawn on the surface of a liquid exposed to the air (or a vacuum), is called its surface tension. Denote that of water by the letter

T. Then the surface tension around the circumference of the inside of the tube is $T2\pi r$, r being the radius of the tube. But this force acting along *the free* surface of the liquid, acts everywhere at an angle $a = 25° \ 32'$ with the vertical surface of the glass. The vertical component, the lifting force, is therefore $T2\pi r \cos a$. If h is the height of the water in the tube above its outside level, the weight of the water lifted will be $\pi r^2 h$, since this is its volume and its density is unity. In equilibrium

$$T2\pi r \cos a = \pi r^2 h$$

$$T = \frac{rh}{2 \cos a}.$$

Clean carefully a glass tube of small bore by nitric acid and a solution of caustic potash and then by thorough rinsing in running water. Dip it in a beaker of water and, holding it vertical, measure by means of a centimeter scale the height h to which the water rises. The inside of the tube should be wet, and to insure this h should be measured just after having slightly raised the tube. It is well to take a number of observations at different parts of the tube. The radius may be determined by drying the tube and filling a length l with mercury. Measure l and weigh the mercury. If w is the weight of the mercury, its density being 13.6

$$w = l\pi r^2 \, 13.6.$$

Whence
$$r = \sqrt{\frac{w}{\pi l \, 13.6}}.$$

With the values thus determined calculate the surface tension of water.

The above discussion of the phenomenon of capillarity applies only to the case of a liquid which 'wets' the solid. A slightly different discussion would be necessary for a liquid like mercury.

16. SIMPLE PENDULUM: FORCE OF GRAVITY.

The absolute unit of force (called a dyne) is such that acting for one second on a mass of one gramme, unconstrained, it will give it a velocity of one centimeter per second. The earth's attraction (gravity) acting alone for one second on a gramme near the earth would give it a comparatively great velocity. If the velocity of a body falling freely were measured at the end of the first second it would be a direct determination of the force of gravity in absolute units. It is difficult to do this accurately. A more ready, though indirect method of measuring the force of gravity, is by the simple pendulum.

The ideal simple gravity pendulum is one in which the swinging body is concentrated at a point and is suspended by a weightless thread. The time of a single swing of such a pendulum depends upon the length of the pendulum and the force of gravity. The relation is expressed by the formula

$$t = \pi\sqrt{\frac{l}{g}}, \text{ or transposing } g = \pi^2 \frac{l}{t^2},$$

where t is the time in seconds of a single swing from one side to the other, or from the centre out to one side and back to the centre again, where l is the length of the pendulum, and where g is the force of gravity on a one gramme mass measured in absolute units.

Suspend a small lead ball by a fine silk thread about 100 cm. long, clamping the upper end of the thread firmly. Measure the length of the pendulum from the point of support to the center of the ball. This is most accurately done by measuring to the top of the ball and then adding the radius, the latter can be measured by a gauge provided with jaws. Place behind the pendulum a screen, and on it make a mark to indicate the central position of the pendulum. By a

table of sines calculate the distances that the pendulum bob must be drawn to one side that it may swing when released over an arc of 5°, 10° and 20° respectively. Mark these distances on the screen. Draw the pendulum bob to one side, and releasing it, allow it to swing over 20°. Determine the time (t) of a single swing. For this purpose either a chronograph and a break circuit clock or an ordinary watch having a seconds hand may be used. If the latter it is desirable that two observers should work together one to count the swings and give the signals which begin and end the count, and the other to observe the watch and record the times. They should exchange positions during the experiment both for the sake of the practice and in order to eliminate as far as possible 'personal' errors from the result. Before beginning the experiment set the minute and second hands of the watch so that they will read even minutes at about the same time. The most sharply defined instant as the pendulum moves to and fro is its transit in front of the index marking the central part of the motion. A single swing of the pendulum is from the transit in one direction until the next transit in the opposite direction, but for convenience the double swing will be timed by counting the transits in one direction only (left to right). The count should be begun by some short, quick signal, for instance the word "tick." In order that the observer who is to record the time may be prepared for the signal, the one observing the pendulum should give signals on the two preceding transits, and before coming to the final signal closing the count, should speak the numbers corresponding to the two preceding transits and then in place of the final number substitute the signal 'tick.' The time of the first and last signal should be recorded in hours, minutes, and seconds and tenths of seconds. The difference between the two divided by the number of swings counted will give the time of a double swing of the pendulum. This halved will be the time (t) of a single swing. It will at first

be difficult to time to tenths of seconds, but with a little practice the estimate will be correct within three tenths. The accuracy with which the time of one swing can be determined is proportional to the length of time over which the whole count is extended. After timing measure again the length (l) of the pendulum to see that it has not changed during the experiment. Substitute these values of l and t in the above equation and calculate the force of gravity (g) in dynes.

Without changing the length of the pendulum swing it through an arc of 10° and repeat the above process. Calculate g.

Again repeat when the pendulum is swinging through an arc of 5°.

Compare the times of a single swing in the three cases. When the arc is varied by this amount can any variation in the time of swing be detected with certainty by the present experiment?

Shorten the pendulum three centimeters and redetermine g, the pendulum swinging through an arc of about 10°.

Replace the lead ball used so far by another of larger diameter and redetermine g, the suspending thread being about 100 cm. long and the arc about 10°.

17. TORSION PENDULUM: MOMENT OF TORSION.

The power of a force to produce rotary motion depends not only upon the magnitude of the force but also upon the perpendicular distance from its point of application to the axis about which the rotation occurs. This rotating power is called the moment of the force, and is equal numerically to the product of the force and the distance. In discussing rotary motion it is convenient to adopt as a unit angle one in which the length of the arc is equal to the radius. As may be readily seen this unit angle is equal to a little over 57°. If one end of a straight wire be twisted, the other end being

rigidly clamped, the wire will exert a force tending to untwist itself. The moment of this force when the end of the wire is turned through the unit angle is called the moment of torsion of the wire.

If a body be suspended by a wire, twisted and then released, it will oscillate with a rotary motion. This constitutes a torsion pendulum, and the formula expressing its various relations is

$$t = \pi\sqrt{\frac{I}{T}}.$$

t is the time of a single oscillation, T is the moment of torsion of the suspending wire, depending on its length, diameter, and material. I is what is called the moment of inertia of the suspended body; throughout the present experiment its value is unknown but remains a constant, depending merely upon the mass, shape, and axis of rotation. It is desired in the present experiment to compare the moments of torsion of different wires.

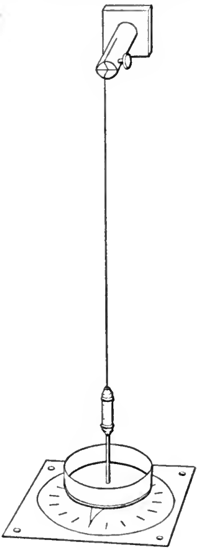


FIG. 10.

Clamp firmly the end of a fine brass wire in the support used in the last experiment. To the lower end of the wire clamp a brass disc and a ring. Fasten to the disc a small index which in passing the zero mark of a graduated circle placed below the disc will indicate the center of oscillation of the torsion pendulum thus constructed. An important modification of the methods of the last experiment is here possible since the oscillations are so slow that there is opportunity to record the time of each of a series of transits. Having set the minute hand of the watch so that it and the seconds hand indicate even minutes at about the same instant, the one observing the pendulum gives the sig-

nal 'tick' as the index passes its central position going from left to right for eleven consecutive transits. The one holding the watch reads and records the exact time of each signal, hours, minutes and seconds, estimating and recording fractions of a second. The average of all these observations will give the time of the sixth transit quite accurately. The pendulum is then allowed to oscillate for about five minutes, and three more signals are given and their times recorded as before. The mean is taken as the time of the middle signal. At the end of about five minutes eleven signals are again given and their times recorded. The mean will be accurately the time of the sixth signal in this set. All the signals must be given when the pendulum is oscillating in the same direction. The times of two transits are now known exactly, the sixth from the beginning and the sixth from the end, but no count has been kept of the number of oscillations made by the pendulum in the meantime. This may, however, be determined as follows: From the times of the first eleven signals determine approximately the time of each oscillation. This can be done by dividing the time between the first and the eleventh signals by ten. Find next the time in seconds between the sixth of the first set of signals and the middle signal of the set of three. Divide this by the approximate time of each observation. It will probably not divide evenly, but near enough to determine to a certainty the number of oscillations between the two. By a similar process find the number of oscillations between the middle signal in the set of three and the sixth signal in the last set of eleven. The sum of the two will be the total number of double oscillations between the sixth transit in the first set and the sixth transit in the last. The time of both these transits is known with special accuracy. Find the difference between the two in seconds and divide by the total number of oscillations, thus determining the time of each double oscillation. Dividing by two will give the time t of a single oscillation. A greater degree

of accuracy might have been attained by continuing the experiment for a longer time, and by observing more transits in the initial and final series.

Adjust the length of the brass wire between the two clamps until it is exactly 100 cm., and determine the time t, when oscillating through an angle of about 90°. Repeat, starting the oscillation by turning the disc through about 180°. Within these limits does the pendulum fail in isochronism by an amount that can be detected with certainty by this method?

Calling T_1 the moment of torsion of the brass wire when 100 cm. long, and t_1 the time of a single oscillation, the formula given above becomes

$$t_1 = \pi \sqrt{\frac{I}{T_1}},$$

shorten the wire until it is exactly 50 cm. long, and retime t_2. If T_2 denotes the new moment of torsion of the wire

$$t_2 = \pi \sqrt{\frac{I}{T_2}}.$$

It is required to find the ratio of T_1 to T_2.

$$\frac{T_1}{T_2} = \frac{t_2^2}{t_1^2}.$$

Substituting the values of t_1 and t_2 calculate the value of the ratio $\frac{T_1}{T_2}$. By an examination of this result deduce the law according to which the moment of torsion of a wire depends upon its length.

Replace the brass wire by a copper wire of the same diameter, a meter long, and retime t_3. By a process similar to the above determine the ratio of the moment of torsion of brass T_1 to the moment of torsion of copper T_3. Reduce to a decimal the fraction expressing this ratio.

It is to be borne in mind that the absolute values of T_1, T_2 and T_3 cannot be determined by the data of this experiment. Only the ratio of one to the other is here sought.

18. TORSION PENDULUM: MOMENT OF INERTIA.

The inertia of a body is that universal property of matter by virtue of which it obeys Newton's law, that at rest it wlll remain at rest, in motion it will continue in motion until acted upon by some external force. The inertia of a body is numerically equal to the force required to give it the unit linear velocity in one second, and this is in turn equal to its mass. Inertia and mass are thus numerically equal. In passing to the consideration of rotary motions not only the force but also the inertia must be regarded as having a certain moment about the axis of rotation. The moment of inertia of a body to rotary motion is numerically equal to the moment of the force required to give it in one second the unit angular velocity. The following discussion will show how this may be calculated. From the definition of the unit angle given in the last experiment it follows that the unit angular velocity is such that every point in the body moves with a linear velocity equal to its distance from the axis of rotation. To start with the simplest case conceive the body to consist of a mass m concentrated at a point, and made to rotate at the end of a weightless arm of length r. When it has the unit angular velocity it will have a linear velocity of r centimeters per second. By definition the force in dynes which must be applied for one second directly to the body to give it this velocity is equal to mr.

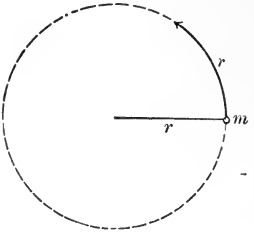


FIG. 11.

The moment of this force is $(mr)\ r$, or mr^2, and this is equal numerically to the moment of inertia of the body. When the body is so large that it cannot be considered as concentrated at a point it must be conceived to be divided into a great number of very small elements. The moment of inertia of the body is equal to the sum of the moments of inertia of the

separate parts, $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$, etc. It follows from this, that the moment of inertia of a radially thin ring, rotating in its own plane about its centre, is equal to its mass multiplied by the square of its radius, since r is the same for every element; and that if I_1 and I_2 are the separate moments of inertia of two bodies their combined moment of inertia is $I_1 + I_2$.

In the torsion pendulum the time of oscillation depends only upon the moment of torsion T of the suspending wire, the moment of the force required to turn one end through the unit angle the other end being rigidly clamped, and upon the moment of inertia I of the suspended body, equal to the moment of the force which acting for one second will give it the unit velocity. The moment of inertia of the ring which rested on the platform in the last experiment can be readily calculated. It is desired, knowing this, to find experimentally the moment of inertia of the platform and clamp, which is of such irregular shape that it cannot be directly calculated, and, knowing then the total moment of inertia of the suspended body, to calculate the moment of torsion of the suspending wire.

Set up the apparatus of the last experiment with a brass wire 100 cm. long. Determine the time of oscillation with the ring off t_1 and with the ring on t_2. Calling I_1 the moment of inertia of the disc and clamp, and I_2 the moment of inertia of the ring;

$$t_1 = \pi \sqrt{\frac{I_1}{T}},$$

and

$$t_2 = \pi \sqrt{\frac{I_1 + I_2}{T}}.$$

$$\frac{t_1^2}{t_2^2} = \frac{I_1}{I_1 + I_2},$$

or

$$I_1 = I_2 \frac{t_1^2}{t_2^2 - t_1^2}.$$

Weigh the ring and determine its mass M. Measure its inside and outside diameter and from this find its mean radius. Calculate the moment of inertia, $I_2 = Mr^2$. Substitute this value in the above equation and determine I_1. Using this value of I_1 calculate the value of T from the first equation.

Remove the ring and slip over the upright rod a brass bar pierced by a hole along one of its shorter axes so that it will rest centrally upon the platform. Retime and determine the moment of inertia I_3 of the bar, regarding I_1 as the known moment of inertia.

SOUND.

19. PITCH BY THE MONOCHORD.

Sound is the audible disturbance communicated to the air, or other surrounding media, by the vibratory motion of the sounding body. Both from a physical and a musical standpoint the pitch of a note is its most important characteristic, and depends upon the number of vibrations per second. If the sounding body is a stretched wire (or string) the number of vibrations per second, when the wire is plucked transversely, depends upon its mass, its length, and the tension with which it is stretched. The material of which it is composed does not affect its pitch, except in so far as its density determines the mass. If l denotes the length in centimeters of the vibrating part of the wire, m the mass in grammes of each centimeter, and t the tension expressed in dynes, then n the number of complete vibrations per second is given by the formula

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}.$$

Since l, t and m can be readily determined, the pitch of the note can be calculated. The instrument for this test, usually called a monochord, consists of a resonating chamber, over which a wire is stretched by a known weight. The length of the vibrating part of the

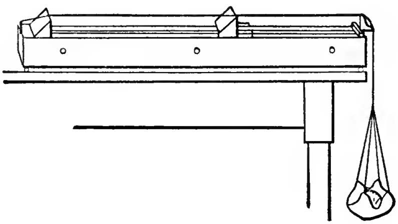


FIG. 12.

the wire may be varied at pleasure by means of the movable fret, until the note is in unison with that emitted by any other instrument. The pitch of the note can then be calculated.

Weigh a steel wire about a meter and a half long, measure its length, and calculate the mass m per centimeter. Stretch it over the monochord by a load of about six kilogrammes, and tune to unison with a tuning fork, varying the length by means of the movable fret. When in unison measure the length l of the vibrating part of the wire. Calculate in dynes the tension with which the wire is stretched, by multiplying the load (in grammes) by the value of g found in Exp. 16, including as part of the load the weight of the pad holding the weights. With these values of l, m and t calculate the pitch number n of the note emitted by the wire by means of the above formula. This will also be the pitch of the fork, since the two are in unison.

Change the load to four kilogrammes and repeat.

Substitute a brass wire for the steel wire and, stretching it with a load of four kilogrammes, repeat.

Average all three values of n. The resulting pitch number of the fork is to be used in succeeding experiments.

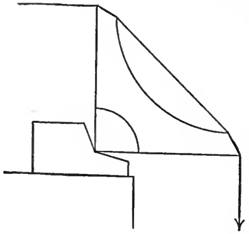


Fig. 13.

If the wire on the monochord is stretched over a right angle lever, in order that the tension in the vibrating part of the wire may equal the load, it is necessary that the vertical and horizontal arms should be of equal length. They are, probably, nearly but not exactly of equal length. The error arising from slight inequality may be eliminated as follows. Tune carefully to unison with the fork and measure the length of the wire. Lift the weights, remove the lever and turn it over so that the arm which was before horizontal is now vertical. Again

tune to unison and measure the length of the wire. This should be repeated several times and the mean of all the lengths taken as the correct value of l to be used in the calculation of n. When the longer arm of the lever is horizontal the tension in the vibrating part is greater than the load; when however the longer arm is vertical the tension is less. Care must be taken throughout the experiment that the lever does not rock so far forward or backward as to rest against one of the shoulders of the recess in which it plays.

In many forms of the monochord the wire is stretched over a second fret and then over a pulley. In this case the tension in the vibrating part of the wire is modified by the friction of the pulley and fret. It is much more difficult to entirely eliminate this error but it may be greatly reduced as follows. Lift the weights and allow them to settle. The friction will in this case diminish the tension. Tune without disturbing the weights and record the length of wire giving unison with the fork. Now draw down on the weights gently, and, letting go, allow them to rise. In this case the friction increases the tension. Retune by varying the length. This should be repeated several times and the average taken of all the lengths thus determined.

When two notes nearly but not quite in unison are sounding together they at one moment reinforce and then oppose each other — the sound is alternately loud and faint. These pulsations of the sound are technically called 'beats,' and furnish a mechanical method of ultimately securing unison. The beats cannot be detected until the wire and the fork are nearly in unison, but when once they are obtained the length of the wire should be varied in such a direction as to make the beats succeed each other more and more slowly until ultimately indistinguishable. If the fork is not mounted on a resonant box its sound may be greatly reinforced by holding its shank on the monochord. In this case the beats may be felt by resting the fingers lightly on the box.

The fork should be set in vibration by bowing, or by striking against a piece of firm leather. If the fork is struck against a hard surface it is liable to become worn and its pitch changed in consequence.

20. MUSICAL INTERVAL: SCALE.

The musical scale consists of a series of eight notes, whose pitch numbers bear to each other comparatively simple ratios. These ratios determine what are called the musical intervals of the notes. When the musical interval of two notes is simple, that is when the ratio of the pitch numbers is a simple ratio, as $2:1$ or $4:3$, etc., the notes when sounded together give a pleasant sensation and are said to be in harmony or in accord. The eight notes constituting the scale are denoted, beginning with the lowest, by the letters *C D E F G A B c*. The musical interval of the eighth note to the first, *c* to *C*, is called an octave, the interval of the fifth to the first, *G* to *C*, is called the interval of the fifth, of *E* to *C* the interval of the third, and so on.

Stretch the steel wire, preserved from the last experiment, over the monochord by a load of about six kilogrammes. Taking all precautions determine by means of this the pitch numbers of each of a set of forks tuned and lettered on the above scale. Having determined the pitch number of each calculate the ratio expressing the musical interval between each note and the lowest note in the scale, *C*. By an examination of these ratios determine to what simple ratios of whole numbers the interval of the octave, the fifth and the third may be reduced. In making the reduction consider one vibration in the pitch number as a possible experimental error. Sound in turn the forks *c*, *G* and *E* with the lower *C* and compare the sensation in each case with the sensation in listening to *B* and *C* sounding together. Reduce the ratio

expressing the interval of the latter combination to as simple a form as possible.

Using the results of the above experiment calculate also the ratio expressing the musical interval between the successive notes, D to C, E to D, F to E, etc. That they may be the more readily compared reduce all the resulting fractions to the same denominator, for example to 1 — in other words express the ratios decimally. Between what notes are the intervals large? When these larger intervals are filled by intermediate notes called sharps and flats the result is called a chromatic scale. In this scale the intervals between successive notes are nearly but not quite uniform.

21. VELOCITY OF SOUND IN AIR BY THE RESONANCE TUBE.

Sound is transmitted as longitudinal vibrations with a definite velocity depending upon the elasticity and density of the medium. This velocity may be measured indirectly in the following manner. Hold the fork whose pitch number n was determined in Exp. 19 immediately over the open end of a large glass tube. The latter should be so arranged that the length of its air column may be varied by a sliding, but tightly fitting, diaphragm. Starting with the air column short, vary its length until it strongly reinforces the sound of the fork. Mark the point of greatest resonance by a narrow rubber band clasping the tube. The impulse sent down the tube by the forward vibration of the fork travels the length l_1 of the tube, and is reflected back in time to reinforce the fork in its backward motion. Thus during the forward motion of the fork, that is during half a vibration, the sound travels twice the length of the tube, $2l_1$. The distance

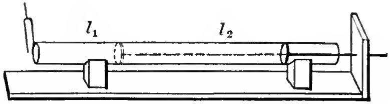


FIG. 14.

actually traveled by the impulse in going each way is a little greater than the length of the tube on account of the reflection from the sides, and the spreading at the open end. The correction to be added is equal to nearly one quarter the diameter d of the tube. During half a vibration therefore the sound goes $2\left(l_1 + \frac{d}{4}\right)$ centimeters, and hence during n whole vibrations, or one second, the sounds goes $4n\left(l_1 + \frac{d}{4}\right)$ centimeters. Whence the velocity of sound in air at the temperature of the room is $V_t = 4n\left(l_1 + \frac{d}{4}\right)$. Continuing to increase the length of the tube find a new position which will also strongly reinforce the fork. By similar reasoning it may be shown that the impulse now travels the length of the tube and back in time to reinforce — not the next — but the next but one backward motion of the fork. The sound therefore travels from the first position of the diaphragm to the second and back again to the first position, $2l_2$, during one vibration. During n vibrations, or one second, it would travel n times as far. Measure the distance l_2 between the two positions of the diaphragm.

$$V_t = 2nl_2.$$

Note the temperature. From these results, calculate the velocity of sound, V_0, in air at 0° C.

$$V_0 = \frac{V_t}{\sqrt{1 + .00366\,t}}.$$

For other intermediate positions of the diaphragm slight resonance of the overtones of the fork may be obtained but they can be distinguished by their higher note. The velocity of sound in air is somewhat affected by the moisture present. If provided with a hygrometer it is well to record the humidity of the air. The change in velocity of sound due

to this will probably not amount to half of one per cent of the whole velocity.

While the air in the longer resonance tube is vibrating with the fork the waves reflected from the closed end of the tube interfere with the succeeding waves coming down the tube at the first position of the diaphragm, causing the air here to remain almost at rest. This point is called a node. The air in the rest of the tube is in vibration to and fro, at a maximum midway between the two positions of the diaphragm and at the open end.

Repeat the experiment using a fork of a different but a known pitch, one from the set used in the last experiment. Within the limits of accuracy of this method does the velocity of sound depend to any appreciable extent upon the pitch of the note?

22. VELOCITY OF SOUND IN BRASS.

A brass rod, held firmly by the middle and stroked with a resined cloth, emits a clear high note by longitudinal vibration. Each half vibrates as does the column of air in the shorter resonance tube of the preceding experiment. Sound, therefore, travels in brass with a velocity sufficient to carry the impulse from the end of the rod to the centre and back, a distance equal to the whole length of the rod, during half a vibration.

Hold a quarter-inch brass rod, about 60 cm. in length, in a horizontal position by a narrow clamp placed in a small vise fixed on the table. Affix to one end of the rod a disc of cork, small enough to slip freely into a large glass tube. Distribute uniformly within the tube, which should be very

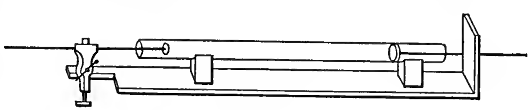


FIG. 15.

dry, a small amount of lycopodium powder. Support the tube on V's on the table, and slip it over the rod. The other end of the tube should be closed by the movable diaphragm. Slowly vary the length of the air column in the tube until it is in resonance with the note emitted by the rod when its free end is stroked. This will be shown by the powder quickly drifting into little heaps or ridges. The ridges mark the points along the tube at which the air is stationary, caused by the interference of the waves reflected from the end of the tube with the waves following down the tube. The distance between these points corresponds to the distance between the two positions of the diaphragm in the preceding experiment, and is therefore the distance that the sound would travel in air during half a vibration. But the sound travels in brass the length of the rod during half a vibration. The velocity of sound in brass, therefore, is to its velocity in air, as the length of the brass rod is to the average distance between the nodes or ridges of lycopodium powder. Calculate the velocity of sound in air, at the temperature of the room, from its velocity in air at 0° C. by the formula $V_t = V_0 \sqrt{1 + .00366\, t}$. Using this and the data above obtained, calculate by the proportion just stated, the velocity of sound in brass.

The clamp may be either a split metal clamp or may be made of a narrow piece of rubber tubing slipped over the rod.

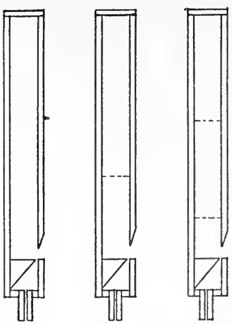
The velocity of sound in any dry gas may be determined by a slight variation of the above experiment. Close the open end of the tube with a cork through which the brass rod can slip loosely and fill the tube with the gas which is to be experimented upon. Allowing a small stream of gas to flow through the tube during the experiment, adjust the position of the diaphragm until the lycopodium powder collects at the

nodes, when the rod is caused to vibrate. The distance between the nodes formed in the gas is the distance that the sound travels during half a vibration of the rod. Comparing this with the distance that the sound has traveled in air in the same length of time in the above experiment, one may readily calculate the velocity of sound in gas, the velocity in air being known.

23. ORGAN PIPES: OVERTONES.

The pitch of a note is determined by the number of vibrations per second, the loudness by the amplitude of the vibrations, and the quality by the number and character of the overtones, faint higher tones sounding with the fundamental note. The organ pipe is perhaps the best subject for the study of overtones. The air in the closed organ pipe when sounding the fundamental note vibrates as does the air in the shorter resonance tube of Exp. 21. The thin sheet of escaping air which strikes against the wedge of the pipe corresponds to the tuning-fork of that experiment, with this difference that it will accommodate its pitch to the length of the pipe, and take up whatever period of vibration the pipe is in resonance for. Thus in a closed organ pipe the vibration of the escaping air will be such that the sound will travel to the diaphragm and back in one half a vibration. If the pipe is shortened the time required for the sound impulse to go to and fro is less, the time of each vibration is less, the number per second greater and the pitch higher. To calculate the pitch n of a closed organ pipe of length l, we can avail ourselves of the following reasoning. The velocity V_t of sound in air at the temperature t of the room can be found from the result of Exp. 21 (see Exp. 22); the time in seconds required by the sound impulse to travel down the pipe and back, $2l$, will therefore be $\frac{2l}{V_t}$. But this is equal to the time in seconds

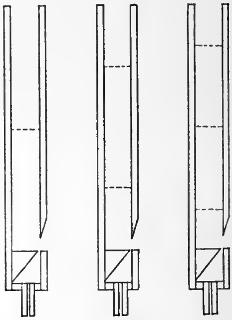
of one half of one vibration, that is one half of the nth of a second $\frac{1}{2n} = \frac{2l}{V_t}$ or $n = \frac{V_t}{4l}$. From the latter formula the pitch n of the pipe may be calculated. A correction is to be applied to l, here as in Exp. 21, for the breadth of the pipe. The correction is in this case large, but too complicated to attempt.



1 2 3

Fig. 16.

Sounding faintly with this fundamental note are the higher overtones. The experiment with the resonance tube when drawn out to its greater length is an example of how a tube may reinforce a note higher than its fundamental note and Exp. 22 with the lycopodium powder is a very instructive illustration of how the air column may still further subdivide. By care and practice in blowing, these overtones may be sounded almost to the exclusion of the fundamental note. Diagrams 2 and 3 represent the manner in which the air column subdivides when sounding the first and second overtones respectively. The dotted lines indicate the position of the nodes at which the air is comparatively at rest and therefore at which lycopodium powder would collect if the disturbance were but sufficiently violent. If the organ pipe is open figures 4, 5, and 6 represent the manner in which the air column subdivides when sounding respectively the fundamental note and the first and second overtones.



4 5 6

Fig. 17.

Measure the length of the organ pipe and record the temperature of the room. From this calculate the pitch of the pipe when closed, using

the value of V_0 obtained in Exp. 21, and neglecting the correction for the breadth and depth of the pipe.

Stretch over the monochord the steel wire, preserved from Exp. 19, by a load of about six kilogrammes. Tuning the monochord to unison with the fundamental (lowest) note emitted by the organ pipe when its end is closed, determine its pitch number. Blowing carefully, sound on the closed pipe the first and second overtones in turn and determine the pitch of each. Calculate the musical interval between each overtone and the fundamental note.

Sound the organ pipe when open, and, tuning the monochord to unison, determine the pitch of the fundamental note and then of the first overtone. Calculate the musical interval between them, and between the fundamental note of the pipe when open, and the fundamental note of the same pipe when closed.

HEAT.

24. CALIBRATION OF A THERMOMETER.

No thermometer can be used for accurate work until it has been calibrated and the errors of its gradations determined. This necessity arises from the possibility of error in the original determination of the 0° and 100° points, from a gradual change in the volume of the glass bulb which would render these marks incorrect even if originally exact, and from the probable variation in the bore of the tube between the 0° and the 100° marks wherefore the degree spaces should be of unequal length at different parts of the stem. In most thermometers the space between the 0° and the 100° marks is divided into 100 equal spaces, leaving the correction for the inequality of the bore entirely to the calibration.

To determine the true zero point pack the thermometer loosely in melting ice. Allow only a sufficient portion of the stem to project to enable one to read the thermometer without removing it from the ice. When the mercury has fallen to its lowest point, and has become stationary, record the reading. This is the true zero point of the thermometer. [In one instance this reading was + 0°.4.]

Place the thermometer in the steam jacket allowing only the upper end of the mercury column to project. Open the upper vent and close the lower so that the steam will circulate freely around the thermometer. When the mercury in the thermometer has reached its highest point, and has become stationary, record the reading. Note also the barometric pressure, and, referring to the table in Appendix, note

the temperature of the steam. [In the experiment above mentioned, the thermometer when in steam at a barometric pressure of 75.5 cm. read 100°.7. The temperature of the steam at this pressure was, from the table, 99°.8. Had the temperature of the steam been 100°, that is °.2 higher, the reading of the thermometer would have been °.2 higher, or 100°.9. This, 100°.9, is therefore the true 100° point on this thermometer.] Calculate the true 100° point — the reading of the thermometer when its true temperature is 100°.

Theoretically the mercury increases in volume as much when its temperature is changed from 0° to 50° as when it is changed from 50° to 100°. To find the true 50° point on the thermometer stem, therefore, we have to find a point such that the volume of the tube from it down to the true 0° point is equal to the volume from it to the true 100° point. Following the directions given below, separate from the mercury in the bulb a column of mercury about 50° long; if one or two degrees greater or less it will be sufficiently near. Place one end of the column of mercury on the true 0° point and read the upper end. Place the upper end of the same column of mercury on the true 100° point and read the lower end. The length of the mercury column may be different but the volume is the same in both cases. Two points, near the centre of the stem are now known, such that the volume of the tube from one to the true 0° point is equal to the volume from the other to the true 100° point. Mid-way between these is the point which divides into equal parts the volume of the tube from the true 0° to the true 100° point. This is therefore the true 50° point. The true 0° and 100° points will probably be fractional readings. As it would be difficult to set the end of the mercury column with accuracy upon a fraction of a degree, set it upon the 0° mark, read the upper end, and to this reading add or subtract as the case may require the amount by which the whole mercury column must be moved that the lower end may coincide with the true 0° point.

Similarly in the second part set the upper end on the 100° mark and read the lower end, and correct this reading as before. [In the case cited above, when the lower end was on the 0° mark the upper end read 49°.1. Had the lower end been on the true 0° point the upper end would have read 49°.5. The upper end being placed on the 100° mark the lower end read 50°.8. Had the upper end been placed on the true 100° point the lower end would have read 51°.7. The mean between 49°.5 and 51°.7 is 50°.6. This is the reading corresponding to the true 50° point.]

The true 25° point divides equally the volume of the tube between the true 0° and 50° points. Separate a column of mercury about 25° long, and proceed as above to determine the true 25° point, in this case working from the 0° and 50° points. By a similar process of interpolation between the 50° and 100° points determine the true 75° point.

From the result thus obtained the correction of the thermometer graduations can be determined for five points on the scale. In each case the correction may be found, with its proper sign, by subtracting algebraically the reading from the true temperature. [In the above case the thermometer read 50°.6. when its temperature was 50°. At this point on the scale the true temperature of the thermometer could be found by subtracting °.6 from the reading. The correction therefore is — °.6 .] Plat the corrections on coördinate paper, taking temperatures for abscissæ.

The following method will generally be successful in securing a mercury column of the desired length. By inclining the thermometer stem downward, and tapping the wrist of the hand holding it upon the palm of the other hand, a column of mercury can be separated from the bulb. If it is too short run it back into the bulb and heat the thermometer in warm water and repeat. If too much separates run it back into the bulb and cool before repeating, or run it into the chamber at the top of the stem, and separate from this point

the desired amount, leaving the rest in the chamber. To start the mercury from the chamber rather violent shaking is required, and corresponding care must be taken to not crush the thermometer.

25. MELTING POINT.

In determining melting points different devices must be used for various substances. If the substance, such as paraffine, changes its color decidedly on changing its state, and if it melts at a low temperature, the following method is a convenient one. Heating a thin glass tube over a Bunsen burner, draw out a capillary tube about a millimeter in diameter. Draw melted paraffine into the tube and allow it to cool. Break the tube and fasten several small pieces by a rubber band to the thermometer, and insert in a test-tube half full of water, and place this in another large vessel containing water (a water bath). Heat the outer vessel. The heat will be communicated slowly to the water in the test-tube. When the temperature of this has reached the melting point of paraffine, the latter will melt and change its creamy appearance, becoming transparent. Take the thermometer reading and apply the correction found from the curve obtained in the last experiment. Withdrawing the test-tube and its contents, allow it to cool slowly, and note the temperature at which the paraffine solidifies. Take the mean as the true melting point. In filling the capillary tubes the paraffine should not be heated far above its melting point.

Dip the thermometer bulb in melted paraffine, and then withdrawing it allow the paraffine which adheres to harden. Place the thermometer in the test-tube, and heat it slowly in the water bath as before. Keeping the water in the test-tube thoroughly stirred, note the temperature at which the paraffine melts.

The following is still a third method. Place sufficient mercury in a test-tube to cover the bulb of the thermometer, and on the mercury float a small piece of paraffine. Dip the bottom of the test-tube in the water bath, and heat slowly. Note the temperature at which the paraffine melts. Withdraw from the water bath, and note the temperature at which it solidifies. Take the mean as before.

26. CUBICAL COEFFICIENT OF EXPANSION OF A LIQUID.

With but few exceptions solids, liquids and gases expand as their temperatures rise. The change in volume of one cubic centimeter of any substance when its temperature is changed one degree is called its cubical coefficient of expansion. By the following method the cubical coefficient of expansion of a liquid may be determined, that of glass being known. For this purpose a flask is needed which is provided with a ground glass stopper perforated by a small capillary opening.

Weigh the flask empty and dry, w. Fill the flask with alcohol and heat in a water bath which is at a temperature slightly above that of the room. As the temperature rises the alcohol expanding more rapidly than the glass will escape through the capillary opening. When no more alcohol escapes the flask and its contents have reached the temperature of the surrounding bath, note this temperature t_1. Remove, and weigh the flask and its contents, w_1. By adding more alcohol remove any air bubble that may have been drawn in. Heat in the water bath to a still higher temperature t_2, about 50°. As the temperature rises the glass expands and thus increases the capacity of the flask. If we denote the initial capacity of the flask by V and the cubical coefficient of expansion of the glass by b, the total increase in capacity of the flask in changing its temperature from t_1 to t_2 has been

$$b\,(t_2 - t_1)\,V.$$

The increase in volume of the alcohol has not only been sufficient to keep pace with the increase in capacity of the flask but also to occasion an overflow. When no more alcohol rises through the stopper, note the temperature t_2, remove the flask, and weigh with its contents, w_2. $w_1 - w_2$ is the weight of the alcohol which has escaped, and divided by the density d is its volume. Therefore the total expansion of the alcohol has been

$$\frac{w_1 - w_2}{d} + b\left(t_2 - t_1\right) V.$$

If the total expansion of the alcohol be divided by its volume, and by the change in temperature, the result will be the increase in volume of each cubic centimeter for each degree change of temperature. Therefore the cubical coefficient of expansion of alcohol is

$$a = \frac{1}{V(t_2 - t_1)}\left(\frac{w_1 - w_2}{d} + b\,(t_2 - t_1)\,V\right),$$

or

$$a = \frac{w_1 - w_2}{d\,(t_2 - t_1)\,V} + b;$$

b the cubical coefficient of expansion of glass is .000025. To determine the density d of the alcohol it is sufficient to divide the weight of the alcohol contained in the flask by its volume, $d = \frac{w_1 - w}{V}$. Substituting this value in the above equation.

$$a = \frac{w_1 - w_2}{(t_2 - t_1)\,(w_1 - w)} + b.$$

Calculate from this equation the mean cubical coefficient of expansion of alcohol.

27. SPECIFIC HEAT BY THE METHOD OF MIXTURE.

The unit quantity of heat is that required to raise the temperature of one gramme of water one degree centigrade. The specific heat of any substance is the quantity of heat, measured in these units, required to raise the temperature of one gramme of the substance one degree centigrade. If two bodies at different temperatures exchange heat without loss or gain from outside bodies the heat gained by the one equals the heat lost by the other.

Make a calorimeter by packing the space between two vessels with wool or cotton. The inner vessel should be closed by a cork through which a thermometer is loosely fitted. Heat some brass (w_1 grammes) in a steam bath until it has reached a stationary temperature t_1 as indicated by a thermometer immersed in it. In another vessel cool some water (w_2 grammes) to a temperature t_2 — nearly zero. Pour quickly, first the water and then the brass, into the calorimeter. Insert carefully the thermometer, and note the temperature t of the mixture. Denote by s the specific heat of brass — that of water being unity.

$$(t - t_2)w_2 = \text{heat gained by the water.}$$

$$s(t_1 - t)w_1 = \text{heat lost by the brass.}$$

The two are equal, provided there is no loss of heat to outside bodies.

$$s(t_1 - t)w_1 = (t - t_2)w_2,$$

$$s = \frac{(t - t_2)w_2}{(t_1 - t)w_1}.$$

From this the specific heat of brass may be calculated.

In order that the loss of heat to the calorimeter may be a minimum it is very desirable that the final temperature of the mixture should be nearly equal to the temperature of

the calorimeter before the water and brass were poured in. It is necessary therefore to perform the experiment tentatively in order to determine the relative quantity of brass and water which should be used. In this preliminary experiment use sufficient water to half fill the calorimeter and enough brass to about one quarter fill it. Both should be weighed. If the rise in temperature of the water is not sufficient to bring it to the initial temperature of the calorimeter more brass should have been used. The quantity of brass required is to that just used about in the ratio of the desired rise in temperature of the water to the rise just obtained. Having in this way determined the quantity of brass which should be used, determine by two experiments the specific heat *s* of the brass. The brass must be dried between each experiment.

An interesting variation would be to determine the specific heat of aluminum.

28. LATENT HEAT OF FUSION.

The latent heat of fusion is the heat required to transform one gramme of a substance at the melting point from a solid to a liquid state without change of temperature. The latent heat of fusion of ice may be determined by placing a known quantity in the calorimeter, pouring over it hot water whose weight and temperature are known and noting the temperature of the mixture when all of the ice is melted. The experiment should first be performed tentatively to determine the relative quantity of ice and hot water which should be used that the final temperature of the mixture may be nearly that of the room. After this preliminary experiment make two final experiments, and in each note the temperature of the hot water, of the calorimeter before pouring in either ice or hot water, and of the mixture when all of the ice is melted; note also the weight of the hot water used, of the ice, and of

the inner calorimeter vessel. The calorimeter vessel is of brass and the specific heat of this material was determined in the last experiment. The weight of the vessel multiplied by its specific heat is what is called its thermal capacity, the quantity of heat required to raise its temperature one degree. Calculate the quantity of heat given up by the hot water, the quantity of heat gained by the water which has resulted from the melting of the ice, and the quantity of heat gained (or given up) by the calorimeter. Knowing these the quantity of heat required to melt the ice can be calculated. From this calculate the latent heat of fusion of one gramme of ice.

While weighing the ice place it upon a cloth which will absorb all that melts. By weighing the cloth after the ice has been taken, the weight of the ice actually used can be found.

29. COOLING CURVE: RATE OF RADIATION.

If the temperature of a cooling body is noted at regular intervals and plotted on coördinate paper, times for abscissæ and the corresponding temperatures for ordinates, the resulting curve is called a cooling curve and is of considerable interest.

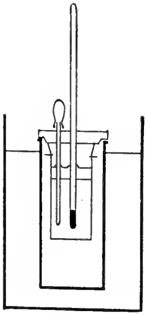


FIG. 18.

Suspend a small nickel-plated vessel, nearly filled with a known quantity (w_1 grammes) of hot water, within a larger vessel, which is in turn immersed nearly to the brim in ice water. The larger vessel should be nickel-plated on the inside so that the two vessels may present polished surfaces toward each other. The two vessels should not touch at any point, but should be as nearly as possible concentric. Through the lid insert a thermometer and stirrer. Keeping the water thoroughly stirred note every minute the temperature as it falls from 70° to 40°. Plot the observations on coördinate paper and through the resulting points draw the cooling curve.

Aside from the loss of heat by the convection currents of air the inner vessel has cooled by direct radiation to the outer vessel. The number of units of heat lost per second in this way is called the rate of radiation. From an examination of the cooling curve determine the time T_1 required for the inner vessel to cool from 70° to 60°. During this change the quantity of heat lost was $(w_1 + c)\ 10$, where w_1 was the weight of the water, c the thermal capacity of the inner vessel and 10, the number of degrees, change of temperature. This loss of heat occurred in T_1 seconds, and therefore the mean loss of heat per second, was $\frac{(w_1 + c)\ 10}{T_1}$ units of heat. This is the mean rate of radiation of the inner vessel in cooling from 70° to 60°, the walls of the surrounding space being at zero. Calculate the mean rate of radiation in cooling from 70° to 60°, 60° to 50°, and from 50° to 40°. Calculate the mean rate of radiation of a nickeled surface per square centimeter by dividing each of the preceding results by the total area of the inner vessel.

This and the succeeding experiment may be performed by dispensing with the outer vessel and the ice and allowing the hot vessel to radiate into the room. In this case thermometers are to be suspended around the walls of the room and read from time to time during the experiment. The difference between the temperature of the hot vessel and the mean reading of all of the thermometers around the room is to be plotted as the ordinates of the cooling curve.

30. RADIATION OF A BLACKENED SURFACE.

The rate of radiation, the loss of heat per second, of a body hotter than the walls of space enclosing it, depends upon the state of its surface (polish, color, etc.) upon the absolute temperature of the enclosing walls and upon the excess of temperature of the hot body. It is proportional to the area

of the exposed surface. In this connection it is interesting to note that the radiating power of a surface, for heat coming from within, is equal to its absorbing power for heat radiated to it from without. Thus a dull black surface absorbs heat readily and is therefore a good radiator. That it absorbs heat readily is evident since so little of the heat radiated to it is reflected.

Repeat the last experiment using vessels presenting blackened surfaces toward each other. The quantity of water, w_1, should be the same as before. Plat the cooling curve, and calculate the mean rate of radiation of the inner vessel when cooling from 70° to 60°, 60° to 50°, and from 50° to 40°. From these three results calculate also the mean rate of radiation per square centimeter of surface between the same ranges of temperature. Similar results for nickel were obtained in the last experiment, and it is now desired to establish a comparison between the two. Write as a ratio the rate of radiation of the blackened surface to the rate of radiation of the nickeled surface when in both cases cooling from 70° to 60°. Treat similarly the results obtained for the range from 60° to 50°, and then for the range from 50° to 40°. In order that these ratios may be more intelligible reduce them all to decimal fractions.

LIGHT.

31. BUNSEN'S PHOTOMETER.

The relative intensities of two sources of light are as the squares of their distances from a screen equally illuminated by each. Of the several methods of testing the equality of illumination the following is perhaps the one most generally in use. When light falls upon a piece of paper having an oiled spot at its center, less light is reflected, and more is transmitted by the oiled than by the unoiled portion of the paper. Viewed from the side next the source of light, therefore, the oiled spot will appear darker than the surrounding paper; viewed from the other side it will appear brighter. If now the opposite side of the paper is equally illuminated, the light lost from one side by transmission is counterbalanced by transmission from the opposite side. The spot would disappear completely when viewed from either side, if it were not for the fact that a certain amount of light is absorbed by the paper, and absorbed unequally by the oiled and unoiled portions. This is the principle employed in the device for comparing two sources of light, known as Bunsen's photometer. Between the two sources of light, which remain stationary at a fixed distance apart, slides a screen of paper having an oiled spot at its center. The whole is in a darkened room. The screen is moved backward and forward until the spot disappears when viewed from the side next one of the lights, and its position on the guides is noted. It is similarly adjusted when viewed from the opposite side, and the new position noted. Midway between these two positions is the point at which the light would have disappeared

when viewed from either side, had there been no absorption of light by the paper. The intensities of the two sources of light are directly as the squares of their distances from this point.

Compare, by means of the photometer, an "adamantine" and a paraffine candle. Before beginning the experiment, cut both candles to such a length that the flame will be about on a level with the screen, and allow them to burn for a moment until the rate of burning becomes steady. Extinguish both candles, and weigh each very carefully with its candle stick. The latter should be provided with a flange to catch the dripping wax. On beginning the experiment proper, note the exact time at which the candles are relighted. While the candles are burning, which should be for fifteen or twenty minutes, adjust the position of the screen at regular intervals, viewing it first from one side, and then from the other. Note the exact time at which the candles are extinguished. Re-weigh the candles carefully. The dripping from the candles should be included in the second weighing.

Determine the mean distance a of the screen from the adamantine candle during the experiment. This mean should include the distance to every position of the screen whether it has been set while observed from the side of the adamantine candle or from the side of the paraffine candle. Similarly determine the mean distance (p) of the screen from the paraffine candle. Knowing the time that the candles have burned and their loss in weight, calculate the rate of consumption of each.

Calculate the relative intensity A of the adamantine candle in terms of the intensity P of the paraffine candle.

$$\frac{A}{P} = \frac{a^2}{p^2}.$$

The intensity of the light given out by a candle in burning depends upon the material of which it is composed, and when

burning at different rates is nearly proportional to the quantity consumed. The standard source of light usually taken as the unit is a paraffine candle burning at the rate of 7.78 grammes (120 grains) per hour. This unit is called "one candle power," often contracted in writing to (C. P.). Calculate the candle power of the paraffine candle as burned in the above experiment in terms of the standard unit. If we denote the rate of consumption of the paraffine candle in the experiment by w_1,

$$P : \text{Standard} = w_1 : 7.78,$$

or

$$P = \frac{w_1}{7.78}\,(C.\ P.)\,.$$

Substitute this value of P in the above equation and calculate the candle power of the adamantine candle;

$$A = \frac{a^2}{p^2}\,\frac{w_1}{7.78}\,(C.\ P.)\,.$$

Denote by w_2 the rate of consumption of the adamantine candle, and calculate what would have been its candle power had it also burned at the rate of 7.78 grammes per hour. The candle power of the adamantine is nearly proportional to the rate of consumption.

The test may be given an even more practical and commercial character by calculating the relative cost per candle power of the adamantine and the paraffine. While the retail prices vary, they are at present twenty-nine cents per kilogramme for paraffine, and forty-three cents per kilogramme for adamantine candles.

A tallow candle may be substituted for the adamantine candle in the above experiment.

An interesting variation would be to determine the cost of kerosene per candle power per hour, when burned in a small lamp. In this case the illuminating power of the flame is to be measured when viewing both its edge and its flat side. This should be regarded as a test of the efficiency of the

lamp, rather than of the kerosene, for the form of the burner greatly affects the illuminating power of the oil.

32. MIRROR, TELESCOPE AND SCALE.

By the first law of geometrical optics, when light falls upon a reflecting surface the angle of reflection — the angle between the reflected light and the perpendicular to the surface — is equal to the angle of incidence. In consequence of this, when the reflecting surface is turned through a definite angle a, the incident light remaining fixed in direction, the reflected light swings through an angle $2a$. This principle is of frequent use in measuring small angles in the device known as the mirror with telescope and scale.

Mount upon the stand of the spectrometer a small, plain mirror — vertical and central. In front of the mirror, at a distance of two or three meters, place a reading telescope, and below it a scale. Raise or lower the telescope, or if necessary the scale, until an image of the latter can be seen in the former by reflection from the mirror. Focus the telescope by varying its length until the image of the scale has sharp definition with the cross hairs. Turn the scale until it is perpendicular to the line joining its center with the center of the mirror. This may be adjusted by making equal the distances to the two ends of the scale from the mirror. Turn the mirror until the central (zero) line of the scale coincides with the cross hairs in the telescope. Take the reading of the graduated circle on which the mirror turns. Turn the mirror through a small angle and again read the graduated circle. The difference between the readings is the angle through which the mirror

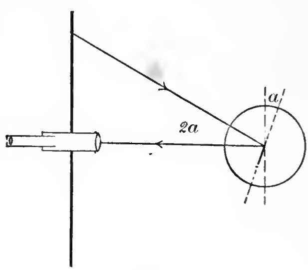


FIG. 19.

has been turned. Take the new reading of the scale as seen through the telescope. The distance from this reading to the zero of the scale, in centimeters, divided by the distance from the mirror to the center of the scale, also expressed in centimeters, is the tangent of twice the angle through which the mirror has been turned. Measure the scale by means of a centimeter scale, and determine the number which when used as a multiplier will reduce all scale readings to centimeters. Measure also the distance from the mirror to the center of the scale. Dividing the scale reading, thus reduced, by the 'scale distance,' calculate the tangent; from the tables find the corresponding angle; dividing by two, determine the angle a through which the mirror has been turned. Compare in parallel columns five angles measured both by the graduated circle and by the mirror, telescope and scale, returning to take a zero reading between each measurement.

If the graduated circle is read by a vernier its principle may be found explained in Exp. 1; if by micrometer microscopes the following is the order of procedure. Focus the microscopes upon the graduated circle, and set the cross hairs sharply upon some line of the graduation. Take the reading of this line. Now turn the mirror to its new position. If the cross hairs coincide again with a line of the graduation, note its reading. The difference between the two readings is the angle through which the mirror has been turned. Very probably the cross hairs will not coincide with a line but will lie beyond by a fraction of a division. The value of this fraction can be determined by interpolation as follows. Turn the micrometer screw back until the cross hairs have traversed this fraction and coincide with the line; record carefully the number of turns and fractions of a turn given the screw. Now turn the screw forward until the cross hairs, having traversed the whole space between two lines, coincide with the line beyond their original position; count as before the number of turns and fractions of a turn given the screw.

The motion of the cross hairs is proportional to the amount of rotation given the screw. Therefore, the fraction being measured is to a whole space, as the amount of rotation given the screw in traversing the fraction is to the amount of rotation given the screw in traversing the whole space. Solving this proportion find the value of the fraction and express it in minutes and seconds. This fraction is then to be added to the reading of the line to which the cross hairs were first brought, before proceeding to find the angle through which the mirror has been moved by subtracting from this reading the original reading of the microscope. Every angle is to be read at the same time by both microscopes, one on either side of the center, to correct for eccentricity of the circle.

33. RADIUS OF CURVATURE OF A CONCAVE MIRROR.

If an object is placed at the center of curvature of a concave mirror the light from it falls upon the mirror perpendicularly, and is reflected back upon its path, coming to a focus again at the center of curvature. Image and object thus coincide. If the object is within the center of curvature its image is without, if without, the image is within. When two objects are at the same distance from the observer they do not appear to move relatively to each other on moving the eye to one side or the other. But when they are not at the same distance, moving the eye to the right or the left will cause the nearer of the two to appear to move in the opposite direction. To illustrate, if we are looking at two vertical rods, one behind the other, when the eye is moved to the right the nearer rod appears to move to the left, and conversely when the eye is moved to the left the rod apparently moves to the right. This phenomenon is called parallax and furnishes a means for determining when the object and its image are at the same distance from the observer.

Place the concave mirror in a suitable frame at the end of a meter rod mounted on a board. On another frame sliding on the rod place two paper millimeter scales back to back, one facing the mirror, the other facing the observer. Adjust by raising or lowering or tipping the mirror until the image of the scale facing the mirror can be seen just over the scale facing the observer. Slide the scales backward and forward until there is no parallax between the image and the scale facing the observer. The center of curvature lies midway between the two scales, if the thickness of the paper separating the two is small. Measure the distance on the meter rod to the lower edge of the mirror, and add a correction for the depth of the mirror and the thickness of the glass. The corrected value is the radius of curvature of the mirror.

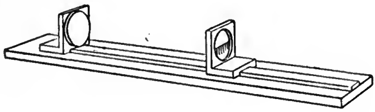


FIG. 20.

When the object is at the center of curvature the image not only coincides with it in position but is, in consequence, of the same size. Using the apparatus as above, adjust until two centimeter lines in the image are exactly a centimeter apart as read on the scale facing the observer. The image, and therefore the center of curvature, now coincides with the scale facing the mirror.

By each method make a number of independent measurements of the radius of curvature.

Having the mirror mounted upon the meter rod as before, turn it toward the window and in front of it place a white screen. Receive the image of some distant buildings upon the screen, and adjust the position of the latter until the image is clearly defined. Measure the distance from the screen to the center of the mirror. This is called the principal focal length f of the concave mirror. Compare the principal focal length and the length of the radius of curvature.

34. CONJUGATE FOCI OF A CONCAVE MIRROR.

If the object be moved either nearer or farther away from the concave mirror than the center of curvature the image will move in the opposite direction. The relative distances of the image and object from the mirror are given by the formula $\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, where p is the distance of the object, q the distance of the image, R the radius, and f the principal focal length — or distance of the image when the object is at an infinite or very great distance. From the property of the exterior angle of a triangle —

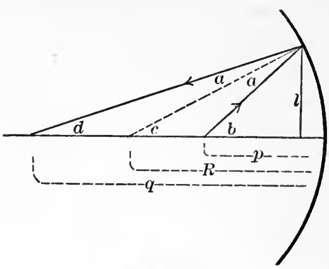


FIG. 21.

$$c = a + d$$

and $$b = a + c\,.$$

Subtracting $$2c = b + d\,.$$

b, c and d are small angles and therefore approximately proportional inversely to their distance from the common subtending line l.

and
$$\begin{aligned} b : c &= R : p \\ d : c &= R : q \end{aligned}.$$

Whence
$$b = \frac{R}{p}\,c \text{ and } d = \frac{R}{q}\,c\,.$$

Substituting in the equation above,

$$2c = \frac{R}{p}\,c + \frac{R}{q}\,c.$$

$$\frac{2}{R} = \frac{1}{p} + \frac{1}{q}.$$

On making p infinite its reciprocal is zero, q for that case is f, and the formula becomes

$$\frac{2}{R}=\frac{1}{f}.$$

Therefore the equation may be written

$$\frac{2}{R}=\frac{1}{p}+\frac{1}{q}=\frac{1}{f}.$$

The approximation in the above mathematical demonstration has a correspondence in the "spherical aberration" of the mirror — the distortion of the image when the aperture of the mirror is large in comparison with its radius.

If the object is moved into the position of the image, the image will be formed at the former position of the object. It is for this reason that the two positions are called *conjugate* foci, and that p and q are called conjugate focal distances.

Place the mirror as before at the end of the meter rod. Mount two paper scales upon separate slides, the one nearer the mirror facing it, the other facing the observer. Starting at the center of curvature, move the object (the scale) toward the mirror at intervals of 5 millimeters. Find for each position of the object the distance of the image by means of the second scale, adjusting it until there is no parallax between it and the image. (The image is no longer of the same size as the object.) Measure in each case p and q from the mirror. Plat the results on coördinate paper, taking p for abscissae and q for ordinates. Calculate f, in several cases, from the values of p and q. Compare with the last experiment.

35. RADIUS OF CURVATURE OF A CONVEX MIRROR.

The following is often a convenient optical method for determining the radius of curvature of a convex mirror, such, for example, as the surface of a convex lens. Place in front of the mirror, at a distance d, a black screen against which slide two white cards A and B. On looking through an aperture at E the cards appear reflected from the mirror at a and b. Since the angle of reflection is equal to the angle of incidence, the perpendiculars to the mirror — that is the extended radii at a and b — will approximately bisect EA and EB respectively. Denote the distance AB by L, ab by l, and the radius by R. Considering the similar triangles having for their common vertex C, and for their bases l and $\frac{1}{2}L$,

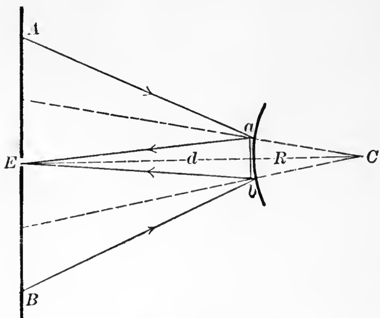


FIG. 22.

$$\frac{R}{l} = \frac{R+d}{\frac{1}{2}L},$$

$$R = \frac{2d\,l}{L-2l}.$$

Place the convex lens on the frame, a meter from the screen. By means of a rubber band fasten on the lens a strip of paper having square ends. Looking through E, move the cards on the screen until the images of their inner edges coincide with the ends of the strip ab. This adjustment may be made more accurately if the images of the cards extend in part above the strip of paper. In this position the

observer can more readily detect when they overlap. Measure *AB*, *ab* and *d*, and calculate the radius of curvature. Repeat with different values of *d*. In this way measure the radii of curvature of both surfaces of a double convex lens. Care must be taken not to be confused by the inverted image seen by reflection from the back surface of the lens. The images are much brighter and more distinct if the cards on the screen are turned toward the light and if a piece of black paper is placed behind the lens.

The radius of a concave mirror may be found in a similar manner. The formula would be slightly different from the above, but may be found through similar reasoning.

36. SPECTROMETER: ANGLES OF A PRISM.

First Method.

Of the two telescopes belonging to a spectrometer, one, called the collimator, is for rendering parallel the rays of light; the

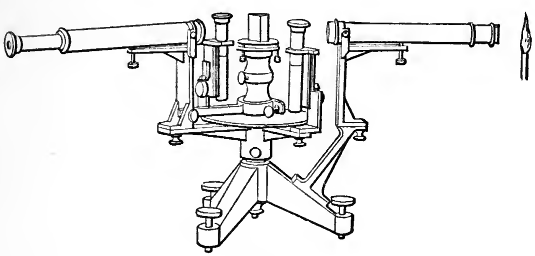


FIG. 23.

other is the observing telescope. The following are the adjustments necessary. Focus the cross hairs in the observing telescope until they can be seen distinctly without straining the eyes. Focus the telescope on some distant object, adjusting until it can be seen sharply with the cross hairs.

and until there is no parallax between them. Turn both telescopes until they point directly toward the center of the instrument, and clamp firmly. Place at a little distance in front of the slit a Bunsen burner (or alcohol) flame colored by common salt or by carbonate of sodium. Darkening the room slightly, turn the observing telescope about the central axis until it is opposite the collimator, and alter the length of the latter until the image of the slit as seen in the observing telescope is in sharp focus and without parallax with the cross hairs. Level, by means of a spirit level, the divided circle, the collimator, and the observing telescope. It is well also at this point to level the central stand or table. The slit in the collimating telescope should be turned quite vertical. If a prism is to be used with the spectrometer its edge should also be vertical. To adjust this, before placing the prism on the central stand, view the slit direct, the two telescopes being opposite each other. Across the front of the slit stretch a fine thread and move it up or down until, as seen in the observing telescope, it coincides with the horizontal cross hair. Place the prism on the stand with one edge turned toward the collimator, and turn the observing telescope until the image of the slit can be seen by reflection from one face of the prism. Level the prism until the image of the thread again coincides with the horizontal cross hair. Repeat when viewing the slit reflected from the other face, and then, turning the telescope back, see that the result of the first leveling has not been disturbed. It will be found most convenient to place one face of the prism perpendicular to the line joining two of the leveling screws and to adjust this face first. The other face of the prism may now be adjusted by means of the third screw alone. By a preliminary trial see that the instrument is so placed that it will be possible to read both sides of the graduated circle in every required position. The slit in the collimator should be narrow. All these adjustments are generally necessary. The graduated circle, as the most deli-

cate and valuable part of the instrument, should be treated with special care.

A spectrometer may be used as a goniometer in measuring the angles of a prism.

Place the prism at the center of the spectrometer with one edge turned toward the collimator. Set the cross hairs of the observing telescope upon the image of the slit reflected from one face of the prism, and read the graduated circle (see the last part of Exp. 32). Turn the telescope until the cross hairs are set upon the image of the slit reflected from the other face. Measure the angle through which the telescope has been turned. It will be readily seen by an inspection of the figure that this angle is twice that of the prism. Having lettered the different angles of the prism so that they can be readily identified, measure all three angles of the prism. Their sum should equal 180°. The divided circle may now be unclamped, turned quarter round, and the work repeated, thus correcting in part for errors of graduation. Care must be taken to not confuse the reflected light with the refracted light which has passed around through the glass.

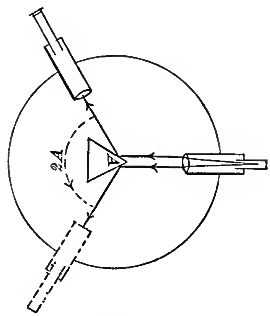


FIG. 24.

Throughout the experiment neither telescope should be touched after the preliminary adjustment. In turning the observing telescope hold as near the solid bearing as possible.

37. ANGLES OF A PRISM.

Second Method.

Clamp the observing telescope permanently near the collimating telescope. Place the prism upon the leveling base so that the image of the slit will be thrown into the observing tele-

scope. Without disturbing either telescope turn the prism by means of the tangent screw until the image coincides with the cross hair. Take the reading on the graduated circle. Turn the prism until the image of the slit is reflected from the next face. Adjust accurately by the tangent screw and again read. The angle through which the prism has been moved subtracted from 180° will be the angle *A* of the prism turned past the telescope. Repeat, using a part of the circle differing by 90° from the preceding. Take the mean. In this way measure all three angles of the prism.

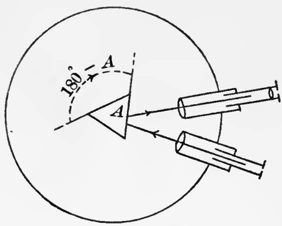


Fig. 25.

38. INDEX OF REFRACTION.

When a ray of light passes from a rare to a denser medium it is bent toward the perpendicular to the surface. For the same substances and the same colored light, the sine of the angle of incidence divided by the sine of the angle of refraction is a constant. It is called the index of refraction, and is usually denoted by the letter n. The index of refraction of light passing from air to glass may be most conveniently determined, when the glass is in the form of a prism, as follows:

Having completed the adjustment outlined in Exp. 36 turn the prism into the position indicated in the diagram. Set the observing telescope upon the refracted image. Rotate the prism, following the refracted ray with the observing telescope. A position can be found at which the light is least refracted. Clamp the support of the prism. Set the cross hairs on the image of

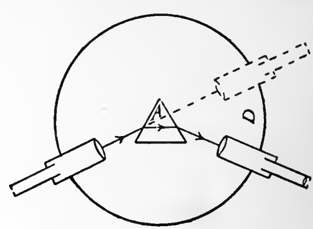


Fig. 26.

the slit, and read the position of the telescope. Remove the prism and set the telescope upon the image of the slit seen directly. The angle through which the telescope has been turned is the angle of deviation of the light, in this case the angle of minimum deviation D.

In the position of minimum deviation the light passes through the prism in a direction such that it makes equal angles with the surface of the glass in entering and in leaving.

$$i = i_1 \text{ and } r = r_1 .$$

Consider the small triangle formed by the three paths of the ray extended. From the property of the exterior angle of a triangle

$$D = (i - r) + (i_1 - r_1) = 2i - 2r .$$

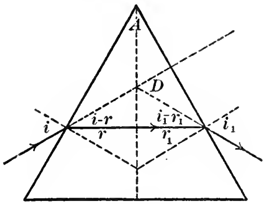


FIG. 27.

From the property of similar triangles each half of A is equal to r.

$$A = 2r \text{ or } r = \frac{A}{2},$$

and substituting

$$D = 2i - A ,$$

$$i = \frac{D + A}{2} .$$

Since

$$n = \frac{\sin i}{\sin r},$$

$$n = \frac{\sin\left(\frac{D + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

The value of A may be taken from the previous experiments.

Measure the index of refraction of the glass for this colored light through all three angles of the prism, using in each case different parts of the divided circle.

39. LAW OF THE DOUBLE CONVEX LENS.

The law connecting the conjugate foci of a convex lens with the radii of curvature of the two surfaces and the index of refraction of the glass is expressed by the formula

$$\frac{1}{p}+\frac{1}{q}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{1}{f},$$

using the notation of Exp. 34. Consider the triangle formed by the path of the ray in the lens and the two radii of curva-

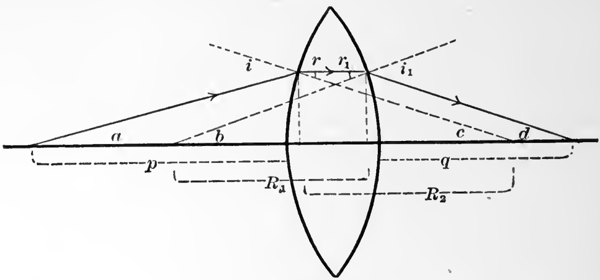


FIG. 28.

ture. Compare this with the triangle formed by the axis of the lens and the same radii. The angles at the intersection of the radii are equal. Hence

$$b+c=r+r_1. \qquad (1)$$

By the property of the exterior angle of a triangle

$a+c=i$ and $b+d=i_1$: adding $a+d+b+c=i+i_1$. (2)

But $n=\dfrac{\sin i}{\sin r}$. Hence for small angles $n=\dfrac{i}{r}$ approximately;

$$\text{or } i=nr, \text{ and similarly } i_1=nr_1.$$

Substituting in (2), $a+d+b+c=n\,(r+r_1)$,

and substituting (1), $(a+d)+(b+c)=n\,(b+c)$.

Transposing and factoring, $(a+d)=(n-1)\,(b+c)$.

Making the same approximation employed in Exp. 34,

$$\frac{1}{p}+\frac{1}{q}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right).$$

Place one of the scales used in Exp. 34 at the end of the meter rod. Mount the lens upon a slide and place it near the

middle of the rod. An image of the scale will be formed upon the other side of the lens. The position of this image may be found by a second scale facing the observer when looking through the lens at the first scale. When there is no parallax between the image and the second scale, read on the meter rod the distances p and q from the two scales to the side of the lens nearest each. Use different values of p and find corresponding values of q. $\frac{1}{f}$ as calculated from each set should be nearly the same. Find f.

Check this value of f as follows: Focus a telescope upon some distant object, adjusting until there is no parallax between the object and the cross-hairs. It is now in focus for parallel rays. Replace the second scale, used above, by the telescope, and with it observe through the lens the distant scale. Adjust the scale until its image is in sharp focus in the telescope. The distance p is now f, the principal focal length of the lens. Taking the lens to a dark part of the room throw upon a screen an image of some brightly illuminated object out of doors. Adjust until the focus is sharp, and measure the distance f from the lens to the screen.

If the lens is the one used in Exp. 35, from the values of R_1 and R_2 there obtained, calculate the value of n — the index of refraction of the glass — by substituting these values in the above equation.

40. MAGNIFYING POWER OF A CONVEX LENS.

The magnifying power of a lens is the ratio of the size of the image to the size of the object.

Place the two scales, mounted as in the last experiment, at opposite ends of the meter rod, and between them find two positions of the lens such that the image of one scale is formed immediately over the other. Note p and q in each case and compare their values.

Place the lens in the position in which it was nearer the scale that was more distant from the observer, and note the magnifying power of the lens — the ratio of the size of the image to the size of the scale immediately below it. Move the lens three centimeters at a time toward the observer, the more distant scale remaining stationary, and follow the image with the other, which may be called the exploring scale. In each case, having adjusted until there is no parallax, read p and q and the magnifying power. Continue until the exploring scale is back to its original position.

Place in parallel columns the ratios of q to p and the magnifying powers, both reduced to decimals. Under what conditions is the total distance between the object and its image found to be a minimum? In this case what is the relation between the conjugate focal distance and the principal focal length, and what is the magnifying power?

Substitute for the exploring scale a sheet of white paper and receive upon it the image of a distant window. Measure the size and distance of the window and of its image, and compare the ratio of the distances with the magnification (fractional).

41. DOUBLE CONCAVE LENS: VIRTUAL FOCUS.

The formula for the conjugate foci of a double concave lens may be proved by a demonstration similar in every step to

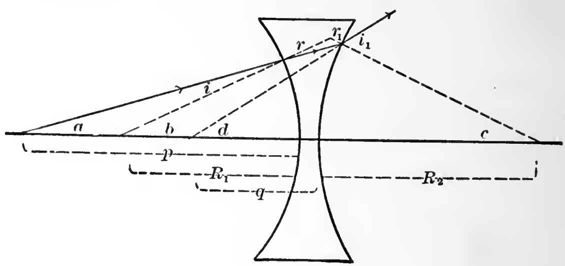


FIG. 29.

that employed in the proof of the formula for the double convex lens.

$$n = \frac{\sin i}{\sin r},$$

whence approximately for small angles :

$$\begin{aligned} i = nr \text{ and } i_1 &= nr_1, \\ b + c &= r + r_1, \\ b - a = i \text{ and } c + d &= i_1, \\ b - a + c + d &= i + i_1, \\ b - a + c + d &= n(r + r_1), \\ b - a + c + d &= n(b + c), \\ d - a &= (n - 1)\ (b + c): \end{aligned}$$

approximately

$$\frac{1}{q} - \frac{1}{p} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right),$$

or

$$\frac{1}{p} - \frac{1}{q} = (n - 1)\left(-\frac{1}{R_1} - \frac{1}{R_2}\right) = -\frac{1}{f}.$$

The latter is the form in which the equation is usually written. Thus if we call p positive, q, and hence also f are in every case negative, meaning thereby that the focus is on the same side of the lens as is the source of light. It is therefore not a real image, but a point from which the light appears to diverge, and hence called a virtual image. The virtual focal distance may be measured as follows :

Having darkened the room place a concave lens at a distance of about a meter from the flame of a small kerosene lamp, turned so that its edge will be presented toward the lens. Against the lens hold a strip of paper by a rubber band, and receive the shadow cast by this upon a white screen at a distance s from the lens. Measure the breadth L of the shadow, and the breadth l of the strip of paper. Measure p, the distance from the lens to the center of the flame, and record the value of s. Let q be the distance from

the lens to the virtual focus — that is, to the point from which the light casting the shadow *appears* to diverge. By inspection of the figure —

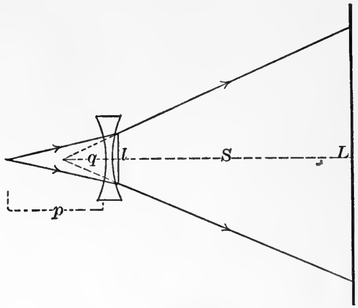


FIG. 30.

$$l : L = q : s + q.$$

Whence $$q = \frac{l\,s}{L - l}.$$

Calculate q. The conditions of the experiment may be varied at pleasure by changing the distance from the lamp to the lens. This may have any value from a fraction of a meter to several meters. For each value of p make several determinations of L with the screen placed at different distances s from the lens, and calculate q. From corresponding values of q and p calculate f. Find the mean of the values of f thus determined.

The value of f may be determined by allowing the direct sunlight to fall upon the lens and measuring the shadow cast upon the screen by a strip of paper across the surface of the lens. The conditions of the experiment may be varied by changing s.

Looking through the concave lens at a long black mark drawn on a sheet of white paper determine which is always further from the lens, the object or the virtual image, and which is apparently the larger. Holding the lens at a little distance from the eye, the mark may be seen over the lens at the same time that the image is seen through it, and their relative distances may be determined by the parallax between them.

Return now to the double convex lens. When the object is within the principal focal length the image is virtual. Examine as above the relative distance and size of an object and its virtual image as seen through a double convex lens.

42. MAGNIFYING POWER OF A TELESCOPE.

The magnifying power of a telescope, its essential feature, is the ratio of the size of an object seen through it to its size as seen by the naked eye. The magnifying power varies with the distance of the object. A rough determination of the magnifying power of a telescope may be quickly made by looking through it at a brick wall, keeping the other eye open. The number of layers of brick seen direct which are required to cover one brick seen through the telescope is the magnifying power.

Having made the above rough determination, place in a vertical position a printed scale of black lines uniformly spaced, and fixing the telescope on its stand, that it may be held steadily, determine its magnifying power by means of this scale, estimating fractions of a division to tenths. Determine in this way the magnifying power when as near the scale as the focusing of the telescope will permit, and at as great a distance as your eyesight, or the arrangement of the room, will permit. Determine the magnifying power also for intermediate points, one meter apart, refocusing the telescope carefully in each position. Two slides moving up and down on the scale will be of great service; the method of using these will readily suggest itself. Plat the results on coördinate paper, platting the distances from the scale to the object lens of the telescope as abscissæ.

43. INVERTING TELESCOPE.

We may construct a simple inverting telescope and examine the functions of the different parts as follows:—

(1) Mount the long focus lens of Exp. 39 upon the holder, and turn toward a window across the room. Receive the image of the window upon a screen of white paper. It will be small and inverted. Remove the screen and examine the

image direct through a short focus lens. It will still be inverted but will be magnified.

(2) Measure the principal focal length of both lenses. Take $O = 10$ meters as the height of an object at a very great distance $P = 1000$ meters. Calculate the distance Q at which the real image I is formed by the object glass; and also the distance p from it at which the eye lens must be placed in order to form the virtual image i at a convenient seeing distance $q = 30cm.$; calculate also the size of this virtual image. The magnifying power of this telescope is the ratio of the angles at the eye lens subtended by i and by O. Since the angles are small they may be measured by their chords $\frac{i}{q}$ and $\frac{O}{P+Q+p}$. Compute. This ratio is the true magnifying power of the telescope. Compare the result with the common, but only approximate, rule that the magnifying power is equal to the ratio of the principal focal lengths.

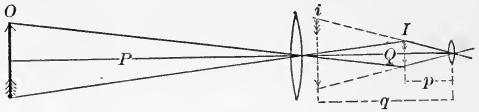


FIG. 31.

(3) Place a scale of broad lines a few meters in front of the telescope thus constructed, and measure directly the magnifying power as in Exp. 42. Compute also this new magifying power, remembering that now the virtual image i is at the same distance as the object O; for the eye looking through the telescope has the same focus as the eye looking at the scale direct.

44. SPECTRUM OF AN INCANDESCENT SOLID: FLAME SPECTRA.

The light produced by common salt (or sodium), when introduced in the Bunsen burner flame, is practically of one color. When this monochromatic light passed through the prism (in

Exp. 38) it was refracted through a definite angle, and when brought to a focus in the observing telescope it appeared as a single line. If however the light passing through the prism had been mixed light of various colors it would have been spread out, technically dispersed, into a colored band or spectrum.

Having carefully focused the observing and the collimating telescope of the spectrometer, and having set the prism for the minimum deviation of yellow light, place in front of the slit a platinum wire and heat to a white heat by a colorless Bunsen burner flame. Examine the spectrum of the incandescent solid, and write a list of the colors in the order of their refrangibility beginning with the least refracted color. A solid of any other material, which will stand the heat without being volatilized, may be substituted for platinum and the result will be the same. Examine the spectrum of the flame of a kerosene lamp. The light is emitted by incandescent particles of carbon.

If the material is volatilized in the flame however, the resulting spectrum has a decidedly different character. Examine the spectrum obtained when the ordinarily colorless flame of the Bunsen burner is tinged by the presence of volatilized strontium (or lithium, calcium or barium) introduced in the form of a salt on a clean platinum wire. Having set the prism for the minimum deviation of the sodium light (it will be impossible to entirely avoid the presence of sodium) measure the angles of deviation of prominent features of the strontium spectrum. Draw carefully to scale a diagram of the spectrum spacing the lines by the angles of deviation, and writing on the side of the diagram the different colors. In this way measure and plat the spectrum of each of the above metals. The metals are best introduced into the flame in the form of the following salts: strontium bromide, lithium carbonate, calcium chloride (or bromide), barium chloride. Dip the platinum wire when red-hot into the

powdered salt and enough will adhere to color the flame brightly. A very small bead of the melted salt is sufficient if skillfully handled. Great care must be taken not to mix the salts.

If, on account of lack of gas, a Bunsen burner flame cannot be secured, the flame of an alcohol lamp may be substituted.

45. SPARK SPECTRA OF METALS.

The spectra of the less volatile metals may be obtained by placing in front of the slit of the spectrometer the spark from the secondary of a Ruhmkorff coil, the terminals, between which the spark passes, being made of the metals under examination. The body of the spark and the brilliancy of the spectrum may be increased by connecting a small Leyden jar around the spark, one coating to each terminal. This will, however, have the effect also of decreasing the length of the spark.

Examine in this way the spark from copper terminals, and, measuring the angles of deviation of the prominent lines, plat the spectrum as in the last experiment.

46. ABSORPTION SPECTRA.

Light is visible, vibratory motion transmitted by the all pervading æther, and the color of the light (corresponding to pitch in sound) depends upon the rate of this vibration. The light from an incandescent solid contains all possible colors, and its spectrum is continuous. If however this light passes through a vapor (or gas) certain parts are absorbed,— lost to the vapor particles, which have a tendency to take up from the light passing through them those rates of vibration which they would themselves occasion if so strongly agitated in burning as to become luminous.

Place the flame of a kerosene lamp in front of the slit of the spectrometer at a distance of twenty-five or thirty centimeters, and between it and the slit hold a test-tube filled with the vapor of iodine. Measure and plat the resulting absorption spectrum. Or : —

Place between the kerosene flame and the slit of the spectrometer a test-tube filled with nitrous acid fumes, obtained by dropping a piece of brass into a very small amount of strong nitric acid at the bottom of the tube. Measure and plat the absorption spectrum.

47. SOLAR SPECTRUM.

If the light of the sun be examined by means of the spectrometer it will be found to yield an absorption spectrum crossed by numerous dark lines. The light coming from the incandescent solid or molten mass has in part been absorbed by the gases and vapors forming the sun's atmosphere. This fact furnishes an exceedingly interesting method of determining the probable chemical composition of the sun.

Reflect the sunlight in a horizontal direction by a plane mirror, and condense it upon the slit of the collimator by a long focus lens. Examine the spectrum and plat the conspicuous features as in the last experiment.

Is there any evidence of the presence of sodium in the sun, sodium being the prominent element of the salt used in Exp. 38 ?

MAGNETISM AND ELECTRICITY.

48. MAGNETIC LINES OF FORCE.

The two points in a magnet from which the magnetic force appears to be directed are called "poles." The unit magnetic pole if placed at a distance of one centimeter from another equal and similar pole will repel it with a force of one dyne. Two poles repel or attract (as they are similar or dissimilar) with a force equal to the product of their strengths divided by the square of the distance between them, and in the direction of the line joining them. The resultant force at any point due to several magnetic poles may be found by the ordinary rules for the composition of forces. A line of force is a line so drawn that everywhere it is in the direction of the resultant force which would be exerted upon a positive pole; or it may be defined as the path which a positive magnetic pole would trace when moving under the magnetic forces alone. Two lines of force never cross.

Place upon the table a small steel magnet and over it a sheet of paper. Sprinkle upon the paper some iron filings, and tap gently. The filings will arrange themselves along the lines of force of the magnet. Do the same with two, three and four magnets, making various combinations of position. Draw these combinations in the note-book, and around them the typical lines of force as indicated by the iron filings. In each case reason out, in a general way, the shape of the curves, so that if called upon you can draw them approximately for any imaginary combination. Observing which poles are positive and which negative mark by an arrow point the direction of each line of force in the diagram, always away from a positive pole and into a negative pole.

49. NORMAL COMPONENT OF MAGNETIZATION.

All the lines of force entering the negative end of a magnet pass through as lines of magnetization, and leave again as lines of force at the positive end; consequently the total positive magnetization is equal to the negative; and at the 'neutral' part, separating the two, the iron is strongly magnetized longitudinally, but gives no indication of it at its surface. It is only that component of the magnetization which is normal to the surface that gives any indication of its presence, for example, by attracting iron filings and small magnets in the neighborhood. A small magnetic needle suspended near will align itself directly toward the bar magnet, and if displaced slightly will oscillate. The time of oscillation depends upon the magnetic force acting upon the needle. A suspended needle can be used in this way to explore the force exerted by a bar magnet at different points, and thus measure the normal component of its magnetization. The force is proportional to the square of the number of oscillations per second.

Fix to the table a brass clamp which holds vertically a steel magnet about 90 cm. long. The clamp should also furnish means for suspending, by a short silk fiber, a small magnetic needle about 4 or 5 cm. north of the long magnet. Place the positive end of the long magnet on a level with the small exploring needle. Set the latter vibrating, and determine the number of vibrations per second. Denote this by n_1. Slip the bar magnet up 6 cm. (or less) and determine the number of vibrations per second, n_2. Repeating until the whole length of the magnet has been traversed, determine n_3, n_4, etc. Note the point at which the exploring needle turns end for end. If we call the magnetic force

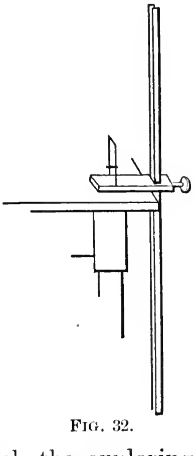


FIG. 32.

positive so long as the positive end of the needle points north, we must call it negative when the needle turns in the opposite direction. The strength of the magnetic force at the different points is proportional to the square of the number of vibrations per second. Plat the results on coördinate paper, taking distances along the magnet as abscissæ, and the squares n_1^2, n_2^2, n_3^2, etc., as ordinates, platting as negative those ordinates for which the needle has turned its positive end south, the force in this case being negative. The resulting curve will represent the force at different points along the magnet due to the earth and magnet together.

In order to get the curve representing the force of the magnet alone, remove the magnet and vibrate the needle under the influence of the earth alone. Determine the number of vibrations per second, n; n^2 is a measure of the earth's force. This is to be subtracted from the positive ordinates, since in these the positive end of the needle was north and the earth was assisting the magnet. It is to be added to all the negative ordinates, since the needle had turned end for end and the earth was opposing the magnet. Both operations are most readily accomplished by raising the axis of abscissæ through a distance equal to n^2. The curve on the new axes will represent the distribution of force near the magnet, and approximately the normal component of magnetization at the different points. Since the total negative is equal to the total positive magnetization, the area of the curve above the new axis of abscissæ is equal to the area below it. Count the number of small squares of the coördinate paper and compare the areas.

50. HORIZONTAL COMPONENT OF THE EARTH'S MAGNETIC FORCE.

Magnetic Pendulum.

The direction of the force which the earth exerts on a north seeking pole is, in general, not horizontal but more or

less dipping. However, most magnetic and electro-magnetic instruments are so constructed that we are concerned only with the horizontal component of this force. The horizontal force exerted on a pole of unit strength is denoted usually by the letter H. If a magnet which can turn about a vertical axis is set east and west, the force on each pole tending to turn the magnet is HQ, Q being the strength of each pole, and H the force on each unit. If the distance from the axis to each pole is d the turning moment of each force is $HQ\,d$. Since the two forces tend to turn the magnet in the same direction, the total moment is $H\,2\,Q\,d$. The factor $2\,Q\,d$ is the moment of the force which would be exerted on the magnet if it were placed in, and at right angles to, a magnetic field of unit strength. It is called the magnetic moment of the magnet, and is usually denoted by the letter M. The moment of the directive force of the earth's field on the magnet may therefore be written $M\,H$. If a suspended magnet be displaced *slightly* from its normal north and south direction and then released it will oscillate to and fro. By analogy with the formulæ of Exps. 16 and 17, it may be shown that if t denotes the time of oscillation and I the moment of inertia of the magnet and support,

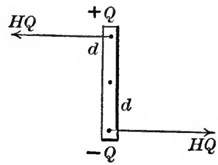


FIG. 33.

$$t = \pi\sqrt{\frac{I}{MH}}.$$

Place a short bar magnet centrally upon the platform support used in Exp. 17, and suspend the latter by a small strand of silk from which all initial twist has been removed by allowing it to untwist with an equal but non-magnetic weight attached. Turn this magnet and support through an angle of about 5°, and allow the combination to oscillate. Time very carefully, $t_1 = \pi\sqrt{\frac{I_1}{MH}}$. Here I_1 is the moment of inertia of

the oscillating mass — the magnet and support. Place in position the brass ring and again time, $t_2 = \pi \sqrt{\frac{I_1 + I_2}{MH}}$. I_2 is the moment of inertia of the ring. The value of the directive moment MH remains the same. Following the method of Exp. 18, eliminate, and find I_1. Substituting this value of I_1 in the first equation, find the value of the product MH. When reduced for this purpose the equation becomes

$$MH = \pi^2 \frac{I_1}{t_1^2}.$$

Throughout the experiment avoid jarring or heating the magnet, or bringing it into contact with other magnets or pieces of iron. Rough handling might vary its magnetization. Preserve the magnet with the same care during the next two experiments.

By the present experiment the product MH has been determined. In the next experiment the ratio $\frac{M}{H}$ will be found, and by eliminating between the two, the values of M and H may be found separately. Of the two, H is the more important.

51. *H* BY THE MAGNETOMETER.

First Method.

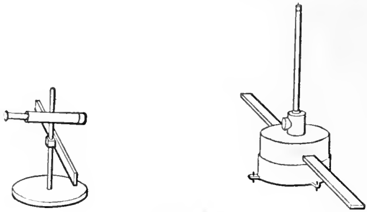


FIG. 34.

The ratio $\frac{M}{H}$ may be found by the deflection which a bar magnet will give a small suspended magnetic needle. The apparatus for this experiment is called a magnetometer. At the center of the instrument is suspended a magnetic needle, the comparatively small deflections of which may be read by means of a mirror, telescope and scale (see Exp. 32).

A long arm on each side of the instrument acts as a support on which may be placed the bar magnet of the last experiment.

Set the apparatus so that the needle will be at the place at which the magnet was suspended in the last experiment. Turn the supporting arm east and west, level the instrument, and adjust the telescope and scale to read zero. Place the magnet of the last experiment on the support with its center at a distance r east of the needle, and with its axis east and west. Read the deflection of the needle. Turn the magnet end for end and again read. Place the magnet at the same distance to the west and repeat. Call the average of these angles a. The force on the north pole of the small compass needle is, if we call q its pole strength, $\frac{Qq}{(r-d)^2}$ attracting, and $\frac{Qq}{(r+d)^2}$ repelling, and the resultant force

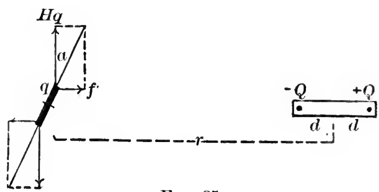


FIG. 35.

$$f = \frac{Qq}{(r-d)^2} - \frac{Qq}{(r+d)^2}, \text{ or } f = \frac{4rdQq}{(r^2-d^2)^2}.$$

We may consider d as being so small that its square may be neglected in comparison with the square of r,

$$f = \frac{4rdQq}{r^4} = \frac{2Mq}{r^3}.$$

The pole is also drawn north by a force Hq. These forces are represented in the adjacent diagram. Since the needle is free to rotate it will set itself with its axis in the direction of the resultant force. Hq and f are at right angles to each other, therefore

$$\frac{f}{Hq} = \tan a.$$

A similar consideration of the forces acting on the south pole of the needle shows that the needle is also in equilibrium as regards these forces when deflected through this angle a.

Substitute for f its value $\frac{2Mq}{r^3}$,

$$\frac{2Mq}{r^3\,Hq} = \tan a.$$

Whence
$$\frac{M}{H} = \tfrac{1}{2}\,r^3 \tan a.$$

Calculate the value of $\frac{M}{H}$. In the last experiment the value of the product MH was determined. Eliminate and find H. The result will be in dynes. Eliminate in a different way and find M.

The experiment should be repeated with the magnet placed at different distances from the suspended needle.

The neighborhood of knives, window weights, and other movable magnets should be avoided.

52. *H* BY THE MAGNETOMETER.

Second Method.

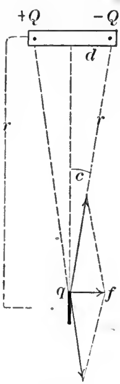


FIG. 36.

Again set the magnetometer in the same place but with the supporting arm north and south. Place the magnet of Exp. 50 north of the needle with its axis east and west, and note the deflection. Reverse the magnet; place it an equal distance to the south; proceed as before to find the deflection of the needle, a. The calculation will be simplified if we assume that the distance r from the center of the bar magnet to the center of the needle is also equal to the distance between their poles. This is only approximately true. The force exerted by each pole of the bar magnet on one pole of the suspended needle is $\frac{Qq}{r^2}$, one attracting and the other repelling. The two (see the figure) do not neutralize each other but yield a resultant east or west, as the case may

be. The component of each in this direction is $\frac{Qq}{r^2}\sin c$, and the total resultant is $f = 2\frac{Qq}{r^2}\sin c$. But from the figure $\sin c = \frac{d}{r}$.

Hence
$$f = 2\frac{Qq}{r^2}\frac{d}{r},$$
or
$$f = \frac{Mq}{r^3}.$$

This is the force drawing the north pole of the needle east (or west), and if it alone acted the needle would turn into this position. But it is also drawn north by a force Hq, and the needle will therefore turn into the direction of the resultant of these two forces. Hq and f are at right angles to each other, therefore

$$\frac{f}{Hq} = \tan a,$$

or substituting for f its value,

$$\frac{Mq}{Hqr^3} = \tan a.$$

Hence
$$\frac{M}{H} = r^3 \tan a.$$

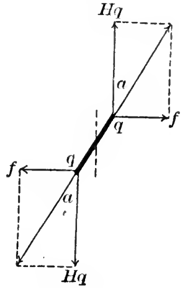


FIG. 37.

Eliminating M between this value of $\frac{M}{H}$ and the value of MH found in Exp. 50, determine the value of H.

Repeat the experiment, using different values of r. If time permits, Exp. 50 should be repeated as a part of this experiment, least the magnetization of the bar magnet has been changed by rough handling. It is not, of course, necessary to redetermine the moment of inertia.

53. TANGENT GALVANOMETER.

When a current of electricity flows through a straight wire it is surrounded by a magnetic field in which the lines of force are concentric circles. Thus the force on a positive magnetic pole would be such as to carry it in a circle around the wire in a direction corresponding to the motion of the hands of a watch facing an observer looking along the wire in the direction of the current. The absolute unit current of electricity is such that if flowing through a wire one centimeter long, bent in the arc of a circle whose radius is one centimeter, it will exert a force of one dyne on the unit pole placed at the center of the circle. If the length of the wire be l the force will be l times as great; if the radius of curvature be r the force will be $\frac{l}{r^2}$, since the force varies inversely as the square of the distance; if the wire make a complete circle the force will be $\frac{2\pi r}{r^2}$; and if there be n turns of wire the force will be $\frac{2\pi rn}{r^2}$ or $\frac{2\pi n}{r}$. This, the force exerted at the center of a coil when the unit current flows through it, is called the constant of the coil, and we shall denote it by the letter G. When a current C flows through the coil the force is C times as great, and if the magnetic pole is of strength q the force is $G\ C\ q$, acting at right angles to the plane of the coil.

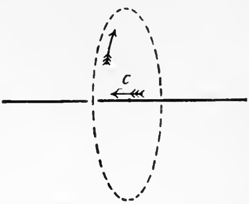


FIG. 38.

If a thin coil of large radius is placed vertically in a north and south plane, and a small magnetic needle hung at its center, the instrument constitutes a tangent gal-

vanometer. Referring to the diagram, and employing the reasoning of the last experiment, the force drawing the north pole of the needle north is Hq, and that drawing it east (or west as the case may be) is the force f, equal to GCq. Since the needle is free to rotate it will turn into the direction of the resultant force, making an angle of deflection, a, such that

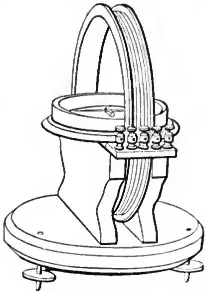


FIG. 39.

$$\frac{f}{Hq} = \tan a,$$

$$\text{or } \frac{GCq}{Hq} = \tan a.$$

$$\text{Whence } C = \frac{H}{G} \tan a,$$

$$\text{or } C = \frac{Hr}{2\pi n} \tan a \left\{ \begin{array}{l} \text{absolute} \\ \text{units.} \end{array} \right.$$

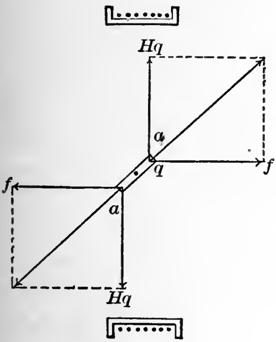


FIG 40.

By means of this formula the current, C, may be calculated in absolute units.

The practical unit of current is called the Ampere, and is equal to one tenth of an absolute unit. The formula therefore for calculating the current in Amperes is

$$C = \left(10 \frac{Hr}{2\pi n}\right) \tan a.$$

The part in parenthesis is called the reduction factor of the galvanometer and will be denoted by the letter F.

Place a tangent galvanometer at the point in the room at which H has been determined, and level carefully. It is essential that the needle shall lie in the plane of the coil when no current is flowing. After having made this adjustment as nearly as possible by eye, complete the adjustment by the following electrical method. Turn the compass until the

pointer on the needle reads zero. Pass a current from a single Daniell cell through the reversing key and through the coil of the tangent galvanometer, using a number of turns of wire, n, such that the deflection will be as near 45° as possible. Take the reading of both ends of the pointer.

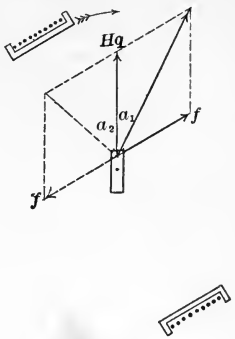


Fig. 41.

Reverse the current and again read. If the deflection is not the same in the two cases the coil is not in the north and south plane. An inspection of the adjacent diagram will show that the coil is to be turned toward the needle (not the pointer) when making the smaller deflection. Each time the coil is turned the compass must be readjusted so that the pointer of the needle will read zero before the equality of the deflections is again tested.

A current from a single Daniell cell is now to be sent around the room through all the galvanometers. Note the deflection in each, reading both ends of the pointer to correct for eccentricity. Repeat after the current has been reversed. Average the four readings. Count the number of turns of wire used in the coil, and determine the radius by measuring the circumference. Calculate the current, using the value of H found in the previous experiments. As the current is the same throughout the circuit, the results furnish a criterion of the accuracy of the individual work.

In using a tangent galvanometer, the experiment should be so arranged, if possible, that the deflection will be in the neighborhood of 45° — between 30° and 60°. This may generally be accomplished by varying n the number of turns of wire. Both ends of the pointer should always be read to correct for possible eccentricity. The wires leading to and from the tangent galvanometer ought always to be twisted about each other, that they may exert no direct influence on

the needle. Whenever the experiment permits, a reversing key should be inserted in the tangent galvanometer circuit, and deflections taken in both directions. If the needle is balanced on a pivot point, the galvanometer should be tapped gently before reading. The jarring will assist the needle in settling to its correct position, overcoming the friction of the pivot. When not in use the needle should be lifted from the pivot.

54. *H* BY THE DEPOSITION OF COPPER.

When an electric current flows through an electrolyte — a compound liquid conductor — the compound is decomposed, part being liberated at the entering or positive electrode, and the other part at the negative electrode. If the liquid is a solution of copper sulphate, .000326 grammes of copper are deposited per second per Ampere on one of the electrodes, and the equivalent amount of acid eats away an equal amount of copper from the other. The weight of copper deposited in a given time is thus a measure of the current which has passed. This, combined with the tangent galvanometer method of measuring the current, gives an electrochemical method of determining H. For if a steady current of electricity has been flowing for T seconds through both the tangent galvanometer and the electrolytic cell, in the former producing a deflection a of the needle, in the latter depositing w grammes of copper, the following two formulæ hold,

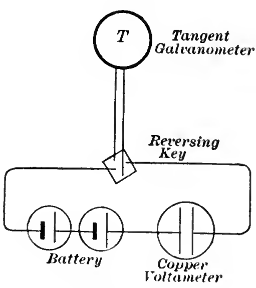


FIG. 42.

$$C = 10\,\frac{Hr}{2\pi n}\tan a,$$

and
$$w = .000326\,TC, \text{ or } C = \frac{w}{.000326\,T}.$$

Since the current is the same in all parts of the circuit,

$$10\,\frac{Hr}{2\pi n}\tan a = \frac{w}{.000326\,T}.$$

Whence $$H = \frac{w\,2\pi n}{.000326\,T\,r\,10\tan a}.$$

From this H may be computed.

The solution in the electrolytic cell should be in the following proportion, 100 parts water, 20 parts sulphate of copper and 5 parts sulphuric acid. The electrodes should be large thin sheets of copper conveniently mounted for handling. After cleaning both electrodes by nitric acid and thorough rinsing in running water, place them in the solution and pass the current from two Daniell cells in series through the tangent galvanometer and the electrolytic cell. Insert in the circuit a reversing key, so placed that it will reverse the current in the tangent galvanometer only. As a preliminary experiment allow the current to flow for four or five minutes, adjusting in the meantime the number of turns of wire in the tangent galvanometer coil so that the deflection will be in the neighborhood of 45°. Remove the electrode upon which the copper has been deposited,—it may be distinguished by its brighter color,—wash in running water and dry. It now presents a bright clean surface for the coming experiment. While handling it avoid touching with the fingers the surface on which the copper has been deposited. Weigh the electrode very carefully and replace it in the cell. Note the exact time at which the circuit is completed. Allow the current to flow at least half an hour. Read the galvanometer every two minutes while the current is flowing. At regular intervals reverse the current through the galvanometer that deflections may be obtained both to the right and to the left. In every case read both ends of the needle. The average of all the readings is the angle a in the above formula. Note the exact time of stopping and the time T in seconds that the current

has been flowing. Remove the receiving electrode and wash it carefully in running water without rubbing, dry and reweigh. The gain in weight is w. Calculate H by the above formula. This is the value of the horizontal component of the earth's magnetic force at the center of the tangent galvanometer, which should be in the same position as during the last experiment.

Using the value of H thus determined recalculate the current measured in the last experiment.

55. EQUIPOTENTIAL LINES AND LINES OF FLOW.

When a current of electricity enters a broad but thin conducting surface it spreads throughout every part of it forming what is called a current sheet. Imaginary lines from one electrode to the other following the direction of the current are called lines of flow. Two points on such a sheet are said to be at the same potential if no current will flow through an outside metallic circuit from one to the other. Lines connecting points all at the same potential are called equipotential lines. Lines of flow and equipotential lines cross everywhere at right angles.

Connect the poles of a Daniell cell with two points upon opposite ends of a sheet of tinfoil which has been pressed upon a plate of hard rubber and ruled into centimeter squares. Touch the sheet at one edge with one terminal a of the astatic needle galvanometer, move the other terminal b up and down on the line two centimeters over. Find a point such that when it is touched by the terminal b the galvanometer shows no deflection. With a remaining in the same position, find a similar point on the line four centimeters over, and so on across the sheet. These points are at the same potential. Locate the points on coördinate paper ruled into similar

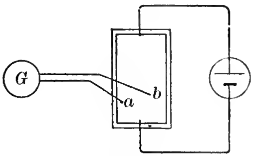


FIG. 43.

squares, and draw a curve connecting them, thus forming an equipotential line. Move the terminal *a* two centimeters along the edge and repeat. In this way find equipotential lines over the whole surface and plat them on coördinate paper.

Draw on the coördinate paper from one pole to the other several lines of flow, the rule being that they cross the equipotential lines everywhere at right angles.

If the galvanometer has several coils they should be joined in parallel. If the needle is too astatic it may be brought under control by means of a small bar magnet placed near it and left undisturbed throughout the experiment.

56. RESISTANCE BY FALL OF POTENTIAL.

When a steady current of electricity flows through a wire, two points along the wire are said to be at different potentials, and it is by virtue of this difference of potential that the current is maintained, overcoming a certain opposition offered by the wire and known as its electrical resistance, R. The law connecting these three quantities, called Ohm's Law, is expressed by the formula $C = \frac{E}{R}$, where E is the difference of potential and C is the current. Hence two wires carrying the same current and having the same difference of potential between ends have also the same electrical resistance.

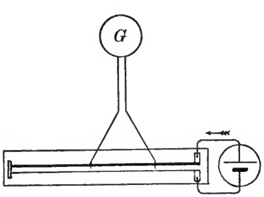


FIG. 44.

Pass the current from a Daniell cell through a brass wire stretched along a meter rod, and back through a german silver wire of the same diameter stretched parallel to the brass wire but without touching it. Touch the terminals of the astatic needle galvanometer, suitably mounted, to two points on the brass wire at a measured distance l apart, about 80 cm. If the deflection of the galvanometer needle is too great — it should be near 45° — reduce the current by shunting the battery, that is, connecting the

poles by a short, thick wire. A controlling magnet can also be used to advantage in reducing the deflection. Transfer the galvanometer terminals to the german silver wire and find two points at a distance l_1 from each other, giving the same deflection in magnitude and direction. The two points on the german silver wire have the same difference of potential as the two points on the brass wire; the same current is flowing in both wires; therefore l cm. of brass wire have the same resistance as l_1 cm. of german silver wire. The resistance per centimeter of the brass wire is to the resistance per centimeter of the german silver wire inversely as these lengths. Express the ratio as a decimal fraction.

Repeat several times, using different parts of the wires and different lengths, also reversing the order of the observations. Good contact must in every case be made by the terminals of the galvanometer on the wire. The current should be allowed to flow for a time before beginning the experiment in order that the action of the battery may become steady.

57. WHEATSTONE'S BRIDGE: WIRE FORM.

If part of a circuit carrying a current is divided there is a continuous fall of potential on both branches from the point at which they separate to the point at which they reunite. For every point on one branch a corresponding point on the other may be found having the same potential. If two such points (m and n) be connected by a wire or "bridge" no current will flow, and if in this circuit a galvanometer be inserted its needle will not be deflected. Denote the resistance of the various parts by the letters a, b, x and r. By Ohm's Law the current in any part is equal to the difference of potential divided by the resistance, and, as

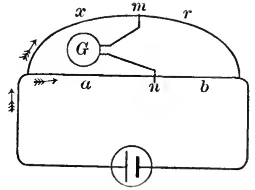


FIG. 45.

no current is carried off by the bridge, the current in b is equal to that in a; hence

$$\frac{\text{the fall of potential in } a}{a} = \frac{\text{the fall of potential in } b}{b}.$$

or

$$\frac{\text{the fall of potential in } a}{\text{the fall of potential in } b} = \frac{a}{b}.$$

Similarly in the other branch, the current in r is the same as in x, and hence

$$\frac{\text{the fall of potential in } x}{\text{the fall of potential in } r} = \frac{x}{r}.$$

Since m and n are at the same potential the fall in a is equal to the fall in x, and the fall in b equals the fall in r. Hence the first members of the last two equations are equal numerator to numerator and denominator to denominator. The second members are therefore equal,

$$\frac{x}{r} = \frac{a}{b} \text{ or } x = \frac{a}{b} r.$$

If the resistance r and the ratio $\frac{a}{b}$ are known x can be calculated. For the branches a and b a simple straight wire can be used, and the ratio of the lengths of a and b may be taken for the ratio of their resistances, since the two are proportional. For r use Ohm coils, the Ohm being the practical unit of resistance.

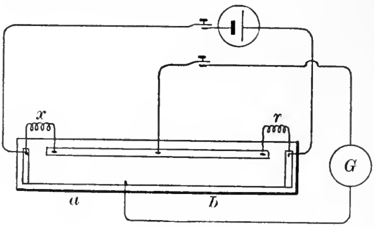


FIG. 46.

Connect a wire bridge as shown in the adjacent diagram and measure the resistance of a number of coils, comparing them with various standard coils, the adjustment being made by moving the sliding contact until the galvanometer shows no deflection when its circuit is closed. A double key should be so inserted that on being

pressed it will close first the battery and then the galvanometer circuit. This will prevent unnecessary waste of the battery. To correct in part for contact resistance at the ends of the meter wire, interchange the known and unknown coils and repeat each measurement. Take the mean of the two measurements. Interchange the galvanometer and battery and repeat, thus illustrating what is called the conjugate property of the bridge.

The more nearly r is taken equal to x the nearer the bridge will be to the center of the wire and the more accurate will be the result. The wire bridge is especially adapted to the measurement of low resistances. The galvanometer should also be of low resistance; if formed of several coils, these should be joined in parallel. The coil whose resistance is to be measured should be so placed that the current flowing through it will not affect the galvanometer directly. This may be tested by disconnecting the galvanometer and making and breaking the current through the rest of the apparatus. In all electrical work the student should bear in mind that good contact can only be secured when the pieces of metal are bright. The wires should therefore be scraped before a joint is made and the binding posts should be occasionally cleaned.

58. WHEATSTONE'S BRIDGE: BOX FORM.

For measuring high resistances the box bridge with a high resistance galvanometer is more desirable than a wire bridge, for in it all the known resistances can be made to approach more nearly the magnitude of the unknown resistance. Here the ratio a to b is definitely fixed by known resistances to any desired ratio, 1 to 10, 1 to 100 or 1 to 1000 at pleasure, and the resistance r is then varied until balance is obtained. The formula is the same as with the wire bridge, $x = r\frac{a}{b}\cdot$ The galvanometer should be of many turns of wire and therefore of high resistance. The conjugate property of the bridge,

the interchangeability of galvanometer and battery connections, was illustrated in the last experiment. It can be proven mathematically or shown experimentally that the best arrangement in this case is that in which the galvanometer connections reach from the junction of the highest two resistances to the junction of the lowest two. The second arrangement, in which the galvanometer and battery have been interchanged, is therefore used.

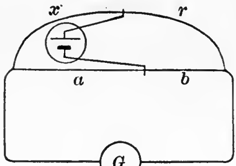


FIG. 47.

Box bridges vary greatly in the grouping of their resistance coils; but by first drawing a diagram of the ideal bridge as above, and also of the resistance box, we may proceed as follows to complete the connections. Choose for a and b that portion of the box the coils of which progress decimally, 1, 10, 100, etc., and for r usually the remainder of the box having the greatest range and capable of the most gradual variation. Consult before each step the ideal diagram, Fig. 45; a and b, if not already connected, should be joined by a plug if possible, if not, by a short, heavy strip of copper; join one end of r to b; to the other end of r join x, and the other end of x to a; where a and b join attach one terminal of the battery, and proceeding in this way complete the connections.

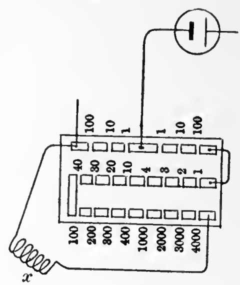


FIG. 48.

Measure carefully the resistance of a coil, starting with a ratio $\frac{a}{b}$ of 1:1, then of 1:10, and if possible of 1:100. It will not generally be possible to secure exact balance, but two values of r may be found differing by one Ohm, which will give

deflections in the galvanometer in opposite directions. The correct value of r lies between. Finally, at the highest ratio of $\frac{a}{b}$ find the values of these deflections in opposite directions, and interpolate for the value of r which would not produce a deflection in either direction.

The taper plugs in the resistance box must be turned in firmly to avoid what is called plug resistance. At the end of the experiment, however, the plugs should be loosened in order to relieve the strain on the cover of the box.

59. SPECIFIC RESISTANCE.

The resistance of a wire varies directly as the length, inversely as the area of the cross-section, and depends upon the material of which it is composed. The specific resistance of a substance is the resistance between opposite faces of a cube of the substance one centimeter square. If we call this R, the resistance r of a wire of cross section a and length l is

$$r = R\frac{l}{a}.$$

Measure with the wire bridge the resistance r of a wire of german silver. Measure its length l in centimeters. The cross section a may be found by weighing a certain length in air and in water; its loss of weight in water is equal to its volume, which divided by the length gives the cross-section. Transposing the above formula,

$$R = r\frac{a}{l}.$$

Knowing r, a, and l, R may be calculated. Find in this way the specific resistance of german silver and of copper. Note in each case the temperature.

Care must be taken that different parts of the bare wire, the resistance of which is being measured, do not touch, and that the wire be not stretched in straightening.

60. TEMPERATURE COEFFICIENT OF RESISTANCE.

The resistance of a conductor depends not only upon its dimensions and material but also upon its temperature. The change of resistance per Ohm per degree change of temperature is known as its temperature coefficient of resistance. Thus, when the resistance of a conductor at some temperature t_1 is known and also its temperature coefficient K, depending upon the material of which it is composed, the resistance at any other temperature t_2, within moderate ranges, can be computed by the formula

$$r_2 = r_1 [1 + K (t_2 - t_1)].$$

Place an openly wound coil of copper wire in a small vessel, and cover the wire with kerosene — a good non-conductor. Connect the coil by short, heavy lead wires to the wire bridge. Surround the vessel with melting ice and when the temperature t_1 of the kerosene has become stationary (nearly 0°) measure the resistance of the coil and lead wires. Now place the vessel in a water bath, and heating the kerosene to a temperature of about 50°, read the temperature t_2 by a thermometer reaching through the cover of the vessel. Again measure the resistance. Disconnect the lead wires from the coil, and joining their ends together, measure their combined resistance. Subtracting this, find the resistances r_1 and r_2 of the coil at the two temperatures t_1 and t_2. By transposing the above equation, or by an immediate consideration of the definition, we have

$$K = \frac{r_2 - r_1}{r_1 (t_2 - t_1)}.$$

From this calculate the temperature coefficient of resistance of copper.

Here, as in subsequent work with the wire bridge, all measurements should be made at both ends of the bridge.

61. RESISTANCE OF COMPOUND CIRCUITS.

When conducting circuits are joined in series (tandem) their total resistance is the sum of their separate resistances,

$$R = r_1 + r_2 + r_3 + r_4, \text{ etc.}$$

When, however, the circuits are joined in parallel, so that the current divides between them, the conductivity, or carrying power, of the whole is equal to the sum of their separate conductivities. But the conductivity of a circuit is the reciprocal of its resistance, and therefore

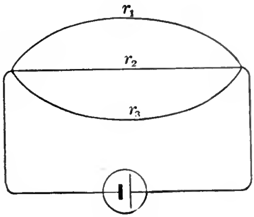


FIG. 49.

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}, \text{ etc.}$$

For two circuits in parallel this becomes

$$R = \frac{r_1 r_2}{r_1 + r_2},$$

for three circuits

$$R = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}.$$

For n circuits the resistance is equal to the continued product of the resistances divided by the sum of the n possible different products formed by taking $n - 1$ resistances at a time.

Measure three resistances separately, and calculate their resistance when in series and in parallel. Place them in a switch board capable of the various combinations, and measure, by means of a wire bridge, their resistances when two are in series and when the two are in parallel, of all three in series and of the three in parallel. Compare the calculated with the measured values.

The lead wires from the bridge to the switch board should be short and heavy, and at the end of the experiment their

resistance should be measured. Their resistance, if appreciable, should be subtracted from all of the results of the above experiment before instituting a comparison with the calculated values.

62. CURRENTS IN DIVIDED CIRCUITS.

When a steady current of electricity divides in flowing through a branched circuit, the current in each branch is such that Ohm's Law holds true: $c = \frac{E}{r}$, or $cr = E$, where in any branch c is the current, r the resistance, and E is the difference of potential of the ends. The potential of the point at which the current divides is common to all the branches separating at that point, and the same is true of the potential where the branches reunite, and hence the difference of potential is the same through all the branches. Denoting this difference of potential by E, and the currents and resistances in the several branches by subscript letters,

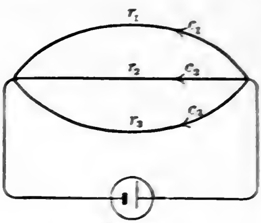


FIG. 50.

$$E = c_1 r_1 = c_2 r_2 = c_3 r_3 = \text{etc.}$$

Hence for any two branches

$$\frac{c_1}{c_2} = \frac{r_2}{r_1}, \qquad \frac{c_1}{c_3} = \frac{r_3}{r_1}, \qquad \frac{c_2}{c_3} = \frac{r_3}{r_2} \text{ etc. (1).}$$

The currents are inversely proportional to the resistances. The sum of the currents in the several branches is equal to the current in the main line. If there are two branches

$$C = c_1 + c_2.$$

If three branches

$$C = c_1 + c_2 + c_3. \text{ (2).}$$

By means of these formulae, knowing the current C in the main circuit and the resistances of the several branches, the current in any branch may be calculated.

Construct a triple branched circuit as shown in the above diagram, including, as parts of r_1, r_2, and r_3, some small resistances, g, one in each branch, and each equal to the resistance of the tangent galvanometer and its lead wires. Insert a similar resistance, g, in the main circuit before it divides.

Removing the resistance, g, in the main circuit and substituting the tangent galvanometer measure the current C. Knowing the resistances of all the branches calculate the current in each, c_1, c_2, and c_3, by combining equations (1) and (2).

Substituting the tangent galvanometer for each of the resistances, g, in turn, measure the current in each branch, and compare with the calculated value.

63. BATTERY RESISTANCE: OHM'S METHOD.

When Ohm's Law, $C = \frac{E}{R}$, is applied to the complete circuit, E is the total electromotive force driving the current, and R is the resistance not only of the metallic portion of the circuit, but also of the battery.

Join in series a Daniell cell, a tangent galvanometer and a resistance box. The tangent galvanometer should be reached through a reversing key, that deflections may be obtained to the right and to the left. Make the resistance in the box one, two, three and four Ohms successively, and measure the currents. So connect the apparatus that, as nearly as possible, the only resistance in circuit besides the cell and resistance box, shall be that of

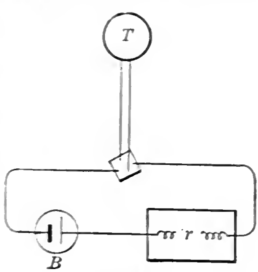


FIG. 51.

the tangent galvanometer and its lead wires g. Denoting by subscript letters corresponding currents and resistances, and the resistance of the cell by B:

$$C_1 = \frac{E}{B + r_1 + g}$$
$$C_2 = \frac{E}{B + r_2 + g}$$
$$C_3 = \frac{E}{B + r_3 + g} \quad \text{etc.}$$

Take the first two of these;—if we assume that E is not changed during the experiment it may be eliminated, giving

$$B = \frac{C_2 r_2 - C_1 r_1}{C_1 - C_2} - g.$$

Pairing in different ways the several experiments compute B—the internal resistance of a Daniell cell.

This method, like most methods for measuring the resistance of a cell, assumes that the electromotive force remains constant when the current flowing through it is changed. This, in general, is not the case on account of a change in what is called the polarization of the cell. This to a certain extent vitiates the value of the results.

64. BATTERY RESISTANCE: MANCE'S METHOD.

In Mance's method of measuring the internal resistance of a cell, the cell replaces the unknown resistance in a Wheatstone's bridge; a key is inserted in the cross branch usually occupied by the galvanometer, and the galvanometer replaces the cell. A current will flow through the galvanometer, but when the adjustment is so made that $\frac{B}{r} = \frac{a}{b}$ the strength of the current flowing through the galvanometer will not be affected by making or breaking the contact at the key. When this adjustment has been made $B = \frac{a}{b}r$ (a formula which is here taken for granted without proof).

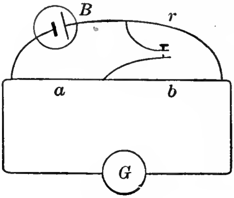


FIG. 52.

Measure the resistance of a Daniell cell by the above method, using first a wire and then a box bridge. The needle of the astatic galvanometer will probably be deflected nearly 90°, but should be brought back by the aid of an outside controlling magnet until the reading is in the neighborhood of zero — until the needle is approximately in the plane of the coil. For this purpose a rather large bar magnet is necessary. It should be so placed that the force which it exerts on the galvanometer needle opposes the force exerted by the current in the coil. Vary a, b, and r, until the galvanometer shows no change in the current flowing through it on making and breaking the circuit at the key. After each change in a, b, or r, it will probably be necessary to readjust the position of the controlling magnet.

This method also is open to the objection that the current flowing through the cell is changed during the experiment, and therefore the electromotive force does not remain constant.

65. BATTERY ELECTROMOTIVE FORCE: OHM'S METHOD.

In the practical system of electrical units, in which the Ampere is the unit of current and the Ohm the unit of resistance, the unit of electromotive force is the Volt. The following is known as Ohm's method of measuring electromotive force.

Join in series a Daniell cell, resistance box, and tangent galvanometer; the latter being reached through a reversing key. Note the deflections in the tangent galvanometer when the resistance in the box is r_1 and when it is r_2, one and two Ohms respectively. Again in another experiment make the resistances three and five Ohms respectively. Calculate the currents in Amperes. If g and B are the galvanometer and battery resistances,

$$C_1 = \frac{E}{B + g + r_1} \text{ and } C_2 = \frac{E}{B + g + r_2}.$$

$$E = C_1 C_2 \frac{r_2 - r_1}{C_1 - C_2}.$$

Calculate the electromotive force of the Daniell cell by each set of observations.

66. ELECTROMOTIVE FORCE, BY COMPARISON.

The following method of "sum and difference" is also sometimes known as Wiedermann's method of comparing electromotive forces. Join in series a Bunsen cell, Daniell cell, resistance box, and tangent galvanometer. Connect the two cells so that they will aid each other, measure the current C_1. Reverse the Daniell cell so that the two cells oppose, and again measure the current C_2. By means of the resistance box the deflections of the tangent galvanometer may be reduced to a convenient size, but the resistance should not be changed during the experiment. Reversing keys, as shown in the diagram, may be convenient. Calling E_1 and B_1 and E_2 and B_2 the electromotive force and resistance of the Daniell and Bunsen cells respectively

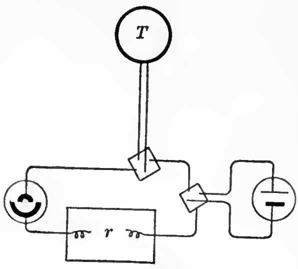


Fig. 53.

$$C_1 = \frac{E_2 + E_1}{B_1 + B_2 + g + r} \text{ and } C_2 = \frac{E_2 - E_1}{B_1 + B_2 + g + r},$$

$$\frac{E_2 + E_1}{E_2 - E_1} = \frac{C_1}{C_2},$$

$$E_2 = \frac{C_1 + C_2}{C_1 - C_2} E_1$$

Thus find the electromotive force of a Bunsen cell in terms of the electromotive force of a Daniell cell. Obtaining the value of E_1 from the last experiment, compute E_2 the electromotive force of the Bunsen cell.

The experiment may be varied at pleasure by changing the resistance.

67. ARRANGEMENT OF CELLS.

When a battery is composed of m similar cells in series, both the total electromotive force and the total internal resistance are m times that of a single cell. If, however, the battery is composed of n similar cells arranged in parallel circuit, the total electromotive force is the same as that of a single cell, but the internal resistance is only $\frac{1}{n}$ that of a single cell. The former arrangement gains by increasing the electromotive force, the latter by decreasing the internal resistance. If a battery is composed of n parallel rows, each row consisting of m similar cells in series, each cell being of electromotive force E and resistance B, the electromotive force of the whole is mE, the resistance of each row is mB, and of the whole battery of n parallel rows is $\frac{mB}{n}$. When from a number of similar cells a battery is to be set up which will give the greatest current through a certain external circuit, the best arrangement is that in which the internal resistance equals as nearly as possible the external.

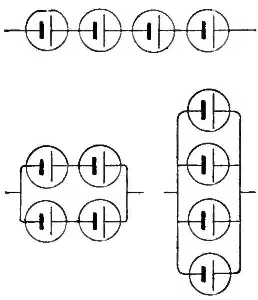


Fig. 54.

Pass the current from a battery of four Daniell cells through a tangent galvanometer and an additional resistance. Make the total external resistance about five Ohms. Arrange the cells — (1) in series, (2) two abreast and two deep, and (3) four abreast, and measure the current flowing in each case. Calculate the internal resistance of the battery each time by

the above rule $\frac{m}{n}B$, borrowing B from the result of Exp. 63. Calculate the current flowing in each case $C = \dfrac{mE}{r + \frac{m}{n}B}$.

taking the value of E found in Exp. 65. Enter the measured and the calculated values of the current in parallel columns, recording, parallel with them, the calculated internal resistances. Repeat, using an external resistance of about one Ohm, and finally with only the galvanometer and lead wires for an external circuit. In each set note which arrangement of cells gives the largest current, and which arrangement has an internal resistance most nearly equal to the external resistance.

68. CALIBRATION OF AN ASTATIC NEEDLE GALVANOMETER.

It is impossible to calculate accurately the reduction factor of an astatic needle galvanometer, since we know neither the directive force on the needle nor the mean radius of the coil. Indeed the deflections do not follow the law of tangents when the length of the needle is great compared with the dimensions of the coil. It is therefore necessary to calibrate the galvanometer with some instrument which is standard, before it can be used for *measuring* currents.

Join a Daniell cell, resistance box, tangent galvanometer and astatic needle galvanometer as shown in the diagram, the two galvanometers being so far apart that they do not influence each other directly. Adjust the resistance R until the deflection in the tangent galvanometer is about 45°. By varying the resistance r the deflection in the astatic galvanometer can be given any desired value. Having measured the current C in the main circuit by means of the tangent galvanometer, the current c through the astatic galvanometer can

be calculated by the principle of branched circuits (see Exp. 62). Start with the resistance r very large, and reduce until the deflection in the astatic galvanometer is about 10°. Note the deflection in both galvanometers, and record the resistance r and R. Reduce r until the deflection in the astatic galvanometer is in turn 20°, 30°, etc., to 60°; repeat the above observations. Calculate the current flowing through the astatic galvanometer in each case, and plat the result on coördinate paper, taking deflections for abscissæ and the corresponding currents for ordinates. In this calculation the resistance of the astatic galvanometer must be taken into consideration, and while measuring it on the Wheatstone's bridge the needle should be removed to prevent its becoming demagnetized.

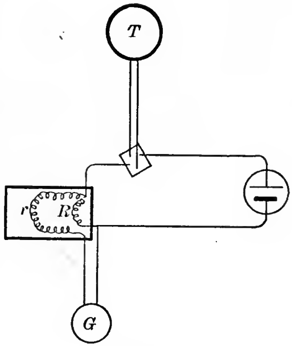


Fig. 55.

69. MEASUREMENT OF RESISTANCE IN ABSOLUTE UNITS.

When two points are so circumstanced that anything, whether running water, magnets, or current electricity, will do work in moving from one to the other, they are said to be at different potentials. When a current of electricity flows through a wire, work is being done, appearing ordinarily in the form of heat. Therefore two points along the wire are said to be at different potentials. They are at the absolute unit difference of potential when the absolute unit of current will do the absolute unit of work per second in flowing from one to the other; and the wire between the two points has the absolute unit of resistance. If C units of current are

flowing between two points whose difference of potential is E, the work done per second is $W = CE$.

Pass the current from two Bunsen cells in series through a spiral of fine german silver wire, wholly immersed in water in the calorimeter of Exp. 27. Include in the circuit, which should be of large wire, a reversing key and a tangent galvanometer. Allow the current to flow, noting at regular intervals the deflection of the galvanometer, until the temperature of the water has risen from t_1 to t_2, about 10 degrees. Record the time — T seconds — required, and the weight w of water. Calling w_1 the thermal capacity of the inner vessel, the quantity of heat generated is $(w + w_1)(t_2 - t_1)$. From the experiments of Joule it is known that 41 600 000 absolute units of work (called ergs) are required to produce one unit of heat. Therefore the heat given to the water must have been generated by

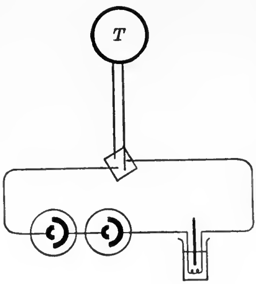


FIG. 56.

$W = 41\,600\,000\,(w + w_1)(t_2 - t_1)$ ergs of work done in the wire. But the work done in the wire per second is CE. and during T seconds

$$W = CET.$$

And since by Ohm's Law $E = CR$, substituting,

$$W = C^2 RT.$$

Equating

$$C^2 RT = 41\,600\,000\,(w + w_1)(t_2 - t_1)$$

or $R = \dfrac{41\,600\,000\,(w + w_1)(t_2 - t_1)}{C^2\,T}$ absolute units of resistance.

Calculate C in absolute units from the tangent galvanometer deflection (See Exp. 53) and substituting in the above equation find R. In this way twice measure the resistance of the coil.

Measure the resistance in Ohms by the wire bridge, and compare the results, and thus obtain the value of an Ohm in absolute units of resistance.

In the equation $C = \frac{E}{R}$ substitute for C the number of absolute units of current in one Ampere, for R the number of absolute units of resistance in an Ohm, and the result will be the number of absolute units of electromotive force in a Volt.

70. THERMOELECTROMOTIVE FORCE.

When a complete circuit is composed of two metals, and one junction is maintained at a different temperature from the other, a current of electricity flows through the circuit. The thermoelectromotive force is proportional to the difference of temperature of the junctions, for ordinary ranges, and depends also upon the metals of which the circuit is composed. If the circuit consists of several strips of the two metals alternating with each other, and if every alternate junction is heated, the combination is called a thermopile, and the electromotive force is proportional to the number of couples. The electromotive force of the thermopile, the difference of potential of its two terminals when the circuit is open, may be measured by balancing it against the difference of potential of two points on a wire through which a current is flowing. This latter may be easily calculated if we know the current c and the resistance r between the two points. If the terminal b of the thermopile is placed in contact with the wire at q, the two have the same potential. We may then find another point p on the wire having the same potential as the terminal a of the thermopile. On joining p and a through the astatic

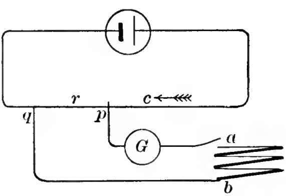


FIG. 57.

needle galvanometer no current flows through it; this being the test of the required adjustment.

In order that apparatus of preceding experiments may be here employed, the arrangement shown in the second diagram may be adopted. The object is to have the current C, through the tangent galvanometer, of such a size that it can be measured, and yet have the current c, through the wire, small. The resistance R should be about twenty Ohms, the resistance R_1 about one Ohm. At the end of the experiment measure the resistance of the wire on the bridge by means of another bridge. This resistance should be known in order to compute from C the branch current c; and from this same measurement can be calculated the resistance r of the wire between the terminals of the thermopile. The different steps of the necessary calculation should by this time be familiar. Having in this way found the total electromotive force of the thermopile, divide it by the difference in temperature of the junctions, and by the number of couples, obtaining thus a coefficient depending only upon the metals of which the thermopile is constructed. If the current is measured in Amperes and the resistance in Ohms, the result is in Volts.

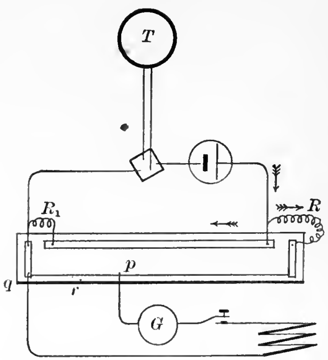


FIG. 58.

A convenient thermopile for this purpose may be made of insulated copper and german silver wires cut into pieces about fifty centimeters long. In the diagram three couples are represented. The terminals of the thermopile should be carefully insulated from each other, one set being immersed in melting ice, the other in steam or boiling water.

71. ELECTROMAGNETISM: INDUCED CURRENTS.

When a current of electricity flows through a coil surrounding an iron core, the latter becomes a magnet with a strength and a direction of polarity depending upon the strength and direction of the magnetizing current.

Pass the current from a Daniell cell through a coil of wire. Bringing near it a small pocket compass, examine the direction in which the lines of force pass through the coil. Slip through the coil a long rod of soft iron until the coil is about its middle. Examine by means of the compass the polarity of the iron. Reverse the current and repeat. Note the relation between the direction of the magnetizing force and that of the induced magnetism.

In previous experiments the galvanometer has been used only to measure steady currents by means of the permanent deflection of the needle, but when properly arranged it may also be used to measure a transient current, which, flowing for an instant through the coil, gives a blow to the needle that sets it swinging. The magnitude and direction of the first excursion of the needle is a measure of the quantity of electricity which has passed, and an indication of its direction. A galvanometer thus used to measure transient currents is called a ballistic galvanometer, and should have, as a rather essential requirement, a needle which is without rapid damping, that is, without being brought quickly to rest by mica vane or copper shell. The rule that the quantity of electricity which has passed is proportional to the angle of the first excursion is near enough correct for present purposes, that it is proportional to the sine of half the angle is, however, more nearly exact.

A preliminary experiment is necessary to enable the observer to determine the direction of the transient current from the direction of deflection of the needle. Shunt off a

small current from a Daniel cell and pass it through the galvanometer. The direction of this current is known, since it flows from the copper to the zinc electrode. Note the binding post at which it enters the galvanometer, and the direction in which the needle is deflected. This will furnish data sufficient to interpret subsequent deflections.

Connect the ballistic galvanometer by long lead wires to a coil consisting of a large number of turns of wire. Thrust into the coil the positive pole of a permanent magnet, observing at the same time the direction of the deflection in the galvanometer. Trace from this the direction in which the induced current flowed around the coil. Remembering the rule given in Exp. 53, reason out the direction of the lines of force which this transient current caused to pass through the coil. Compare this with the direction of the lines of force thrust into the coil by the motion of the permanent magnet (see Exp. 48). Withdraw the magnet and compare the lines of force through the coil, due to the induced current, with the lines of force which are being withdrawn with the magnet. Test the generality of these conclusions by the subsequent experiments, noting also the magnitude of the deflections.

Before each experiment the needle of the galvanometer should be brought to rest. This can best be done by skillfully presenting to it, and then quickly withdrawing, a weak permanent magnet. Throughout the experiment the galvanometer should be so far away from the rest of the apparatus that its needle may not be influenced directly by the magnets being tested.

(3) Present the negative pole of the strong permanent magnet to the coil, as in the above experiment with the positive pole; (4) withdraw. (5) Present the positive pole to the opposite side; (6) withdraw. (7) Place against this coil another coil, through which a strong current is flowing (noting its direction); (8) withdraw. Present again, but without any current flowing through it, and then (9) make

the current; (10) break the current. Make the current, and then (11) reverse it. Place through the two coils a long, soft iron rod and (12) break the primary current, as it is called; (13) make in the same direction as before; (14) break; (15) make in the opposite direction. The difference between the magnitudes of (14 and 15) is due to residual magnetization. (16) Reverse the current.

Vary these experiments until you are so familiar with the phenomena that, the conditions being given, you can predict the direction of the induced current.

72. DISTRIBUTION OF MAGNETIZATION.

The throw of the galvanometer needle when used as in the last experiment is a measure of the change of magnetic induction through the coil. Sliding the secondary coil, in circuit with the ballistic galvanometer, along to different parts of the iron rod, and reversing the current through the primary which has remained at the middle of the rod, we may determine, approximately, the relative longitudinal magnetization at different points.

Ordering the experiment as you may see fit, determine the magnetization at different points along the iron rod.

Plat the results on coördinate paper and compare the curve with the curve for the normal component of magnetization obtained in Exp. 49.

APPENDIX.

I. SIGNIFICANT FIGURES.

An investigator, who, in publishing the results of his experiments, carelessly keeps a greater (or a less) number of figures than are known, not only deceives himself, but often misleads others in regard to the accuracy to which he has attained. There are other methods of discussing results, but the following is sufficient for the purposes of the present course. It may perhaps be best illustrated by taking the first experiment as an example.

The measurement of the dimensions of a cylinder by a vernier gauge gives as its length 5.215 cm., and as its diameter 3.020 cm. The vernier gauge cannot measure accurately thousandths of a centimeter, and the last figure in each of the above values is in great doubt. The true length and the true diameter of the cylinder may be either greater or less than these values by .002 of a centimeter. To calculate the volume of the cylinder we have the formula $V = l\pi r^2$. The radius is 1.510. The arithmetical work is given in full as it is most instructive.

$$
\begin{array}{r}
1.510 \\
1.510 \\
\hline
\mathbf{0000} \\
151\mathbf{0} \\
755\mathbf{0} \\
151\mathbf{0} \\
\hline
2.28\mathbf{0100}
\end{array}
$$

Since the last figure in the value of the radius, the cipher, is in doubt, all of the figures printed in heavy faced type are in doubt. The first cipher in the product is in doubt, therefore the succeeding figures are wholly unknown, and should be discarded. Proceeding in the calculation.

$$\begin{array}{r}
2.280 \\
3.141 \\
\hline
2280 \\
9120 \\
2280 \\
6840 \\
\hline
7.161480
\end{array}$$

Again discarding the figures which are wholly unknown. and keeping only the first figure which is in doubt,

$$\begin{array}{r}
7.161 \\
5.215 \\
\hline
35805 \\
7161 \\
14322 \\
35805 \\
\hline
37.344615
\end{array}$$

The final result is to be entered as 37.34, the succeeding figures, which are wholly unknown, being discarded. That there has been a great saving of labor will be obvious if the student will but repeat the above calculation without discarding any of the figures.

If the calculation is by logarithms this method of discussing the results is not practicable, and the following method may be employed. If the calculation involves only multiplying and dividing, the percentage error introduced into the result by any factor is equal to the percentage error in that factor. Thus, in the above example, the error which could pass undetected in the measurement of the length, is supposed to be .002 cm.; this is .04 per cent. of 5.215. Therefore the possible error in the result arising from this source is .04 per cent. of 37.3446 . . . or .014 of a cubic centimeter. Obviously therefore the first figure 4 in the result is slightly in doubt. and the subsequent figures are unknown and must be dis-

carded. If one of the factors is squared the percentage error in the result is twice as great. If a factor is cubed its error is tripled in the result. If the square root of a factor is taken the possible error is halved in the result. Thus in the above calculation of the volume of the cylinder the radius is squared. The error in the measurement of the diameter which could pass undetected is assumed to be .002 cm. This is nearly .07 per cent. of 3.020 — the diameter. Since the radius is squared the possible percentage error arising in the result from this source is .14 per cent.; .14 per cent. of 37.3446 cu. cm. is .05 cu. cm. Thus the first figure 4 in the result is in doubt, subsequent figures are unknown, and the result should be written 37.34.

II. GRAPHICAL REPRESENTATION.

When a series of results depends upon some one regularly varying factor in the experiment a very instructive graphical representation may be made by platting the results on coördinate paper. This method of recording results may be most simply illustrated by an experiment in which the temperature of a cooling body has been observed at frequent intervals: in this the temperature observations form the series of results, depending upon the time which has elapsed as the one varying condition. In the following the first column of figures gives the time in minutes which had elapsed from the beginning of the experiment, and the second column the corresponding temperature observations.

Time.	Temperature.
2. min.	73.° C.
6.	65.°
8.	58.°
13.	51.°
18.	45.°
24.	38.°

etc.

On the coördinate paper which is ruled by numerous equally spaced lines, a certain horizontal line is chosen as the "axis of abscissæ" and a certain vertical line as the "axis of ordinates"; the intersection of the two lines is called the origin. A horizontal distance measured along the axis of abscissæ is to represent the time from the beginning of the experiment to a certain observation, and a vertical distance measured from the point thus reached is to represent the corresponding temperature observations, each according to some chosen scale. Each observation is to be platted in this way and a curve drawn through the resulting points. In the curve platted below, each horizontal space represents 5 minutes, and each vertical space represents a change of temperature of 10 degrees.

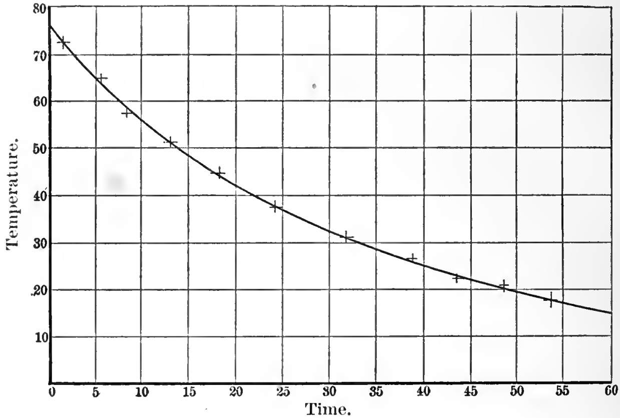


Fig. 59.

It is not always desirable, or possible, to draw the curve through all of the points marked, but if it is known that the curve should be a smooth curve, and if the points are numerous, it should be drawn between them thus averaging, in a measure, the errors of observation.

If the quantity platted is large in comparison with its variations, the scale on which the curve is platted must be correspondingly large in order that it may show these varia-

tions. This may be done without using an awkwardly large sheet of paper by platting only the curve and dispensing with the paper below. This is illustrated in the following curve, representing the load required to sink a Nicholson's hydrometer in water at various temperatures.

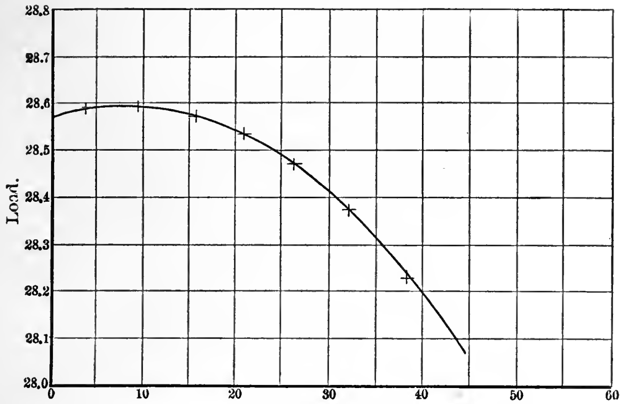


FIG. 60.

The scale on which the curve is drawn should always be indicated along the two axes.

If a quantity is being platted as ordinates some of whose values are positive and some negative, the positive values are platted upward from the axis of abscissæ, the negative values downward. Thus in the first example if the body had cooled below zero, the curve would have crossed the axis of abscissæ. When the abscissæ are negative they are to be platted off to the left from the origin.

Curves are of especial service as quick and accurate means of interpolation. Thus, from the curve, the temperature of the cooling body fifteen minutes after the beginning of the experiment was 48° C.; the weight required to sink the hydrometer to the given mark on the stem in water at the temperature of 19° C. would be 28.55 grammes.

III. TABLES.[1]

1. *Weight in grammes of one cubic centimeter of dry air at different temperatures and barometric pressures.*

(NOTE. — Two ciphers are to be prefixed to all the numbers in the body of the table.)

Temperature (centigrade)	Barometric Pressure[2] (centimeters of mercury).							
	70 cm.	71 cm.	72 cm.	73 cm.	74 cm.	75 cm.	76 cm.	77 cm.
10°	.001149	1165	1181	1198	1214	1231	1247	1263
11°	1145	1161	1177	1194	1210	1226	1243	1259
12°	1141	1157	1173	1189	1206	1222	1238	1255
13°	1137	1153	1169	1185	1202	1218	1234	1250
14°	1133	1149	1165	1181	1197	1214	1230	1246
15°	1129	1145	1161	1177	1193	1209	1225	1242
16°	1125	1141	1157	1173	1189	1205	1221	1237
17°	1121	1137	1153	1169	1185	1201	1217	1233
18°	1117	1133	1149	1165	1181	1197	1213	1229
19°	1113	1129	1145	1161	1177	1193	1209	1224
20°	1110	1125	1141	1157	1173	1189	1204	1220
21°	1106	1121	1137	1153	1169	1185	1200	1216
22°	1102	1117	1133	1149	1165	1181	1196	1212
23°	1098	1114	1130	1145	1161	1177	1192	1208
24°	1094	1110	1126	1141	1157	1173	1188	1204
25°	1091	1107	1122	1138	1153	1169	1184	1200
26°	1088	1103	1118	1134	1149	1165	1180	1196
27°	1084	1099	1114	1130	1145	1161	1176	1192
28°	1080	1095	1110	1126	1142	1157	1172	1188
29°	1077	1091	1107	1122	1138	1153	1169	1184

[1] Taken from Kohlrausch's Physical Measurements, Whiting's Physical Measurements (which contains very complete and excellent tables), and from the tables of Landolt and Börnstein.

[2] In making barometer readings it is customary to read to the top of the curvature of the mercury, and this is sufficiently accurate for the purposes of

If the air is not dry a correction must be applied to the above values. The presence of the vapor of water as a part of the atmosphere decreases the weight per cubic centimeter. For various dew points the values of this correction are as follows:

Dew Point.	Subtract.
0°	.000 003
5°	.000 004
10°	.000 006
15°	.000 008
20°	.000 010
25°	.000 014

The main use of the above table is in calculating the buoyancy of the air. If the object being weighed does not occupy over 100 cubic centimeters of space, and if the weighings have not been carried further than milligrammes the above correction for the humidity of the air can usually be neglected.

2. *Space in cubic centimeters occupied by one gramme of water at different temperatures.*

0°	1.000 12	10°	1.000 26	20°	1.001 73
1°	1.000 07	11°	1.000 35	21°	1.001 94
2°	1.000 03	12°	1.000 46	22°	1.002 16
3°	1.000 01	13°	1.000 57	23°	1.002 38
4°	1.000 00	14°	1.000 70	24°	1.002 62
5°	1.000 01	15°	1.000 85	25°	1.002 87
6°	1.000 03	16°	1.001 00	26°	1.003 13
7°	1.000 07	17°	1.001 16	27°	1.003 39
8°	1.000 12	18°	1.001 34	28°	1.003 67
9°	1.000 18	19°	1.001 53	29°	1.003 95

the present course, but it is well to bear in mind that when greater accuracy is to be obtained corrections must be applied for the depression by capillarity, for the temperature of the mercury and of the measuring scale, and for the force of the earth's attraction at the particular locality in which the experiment is being performed.

3. *Boiling point of water under different barometric pressures.*

Barometric Pressure.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
70.	97.72	97.76	97.80	97.84	97.88	97.92	97.96	98.00	98.03	98.07
71.	98.11	98.15	98.19	98.23	98.27	98.31	98.34	98.38	98.42	98.46
72.	98.50	98.54	98.58	98.61	98.65	98.69	98.73	98.77	98.80	98.84
73.	98.88	98.92	98.96	98.99	99.03	99.07	99.11	99.14	99.18	99.22
74.	99.26	99.30	99.33	99.37	99.41	99.44	99.48	99.52	99.56	99.59
75.	99.63	99.67	99.71	99.74	99.78	99.82	99.85	99.89	99.93	99.96
76.	100.00	100.04	100.07	100.11	100.15	100.18	100.22	100.26	100.29	100.33
77.	100.36	100.40	100.44	100.47	100.51	100.55	100.58	100.62	100.65	100.69

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W. R. McDaniel, *Prof. of Mathematics, Western Maryland College.* Hardy's Calculus and Analytic Geometry are certainly far better books for the college class-room than any others I know of. The feature of both books is the directness with which the author gets right at the very fact that he intends to convey to the student, and the force of his presentation of the fact is greatly augmented by the excellent arrangement of type and other features of the mechanical make-up.